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# Performance Modeling for Two-hop Relay with Erasure Coding in MANETs

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**Abstract**—Among the “store-carry-forward” kind of protocols, the two-hop relay and its variants have become a class of attractive routing protocols for the mobile ad hoc networks (MANETs) due to its efficiency and simplicity. This paper focuses on the performance modeling for two-hop relay with erasure coding, a promising technique for improving the delay performance of conventional two-hop relay with simple replication. A general Markov chain-based theoretical framework is first developed to model the complicated message delivery process in such a network, based on which not only the mean value but also the variance of message delivery delay are derived analytically. The important medium contention, interference and traffic contention issues are carefully incorporated into our analysis, so the new theoretical framework can be used to precisely predicate the message delivery delay performance of two-hop relay with erasure coding, as verified by extensive simulation results.

## I. INTRODUCTION

Among the “store-carry-forward” kind of protocols, the two-hop relay has become a class of attracting routing protocols for the mobile ad hoc networks (MANETs) due to its efficiency and simplicity [1]. In the two-hop relay routing, the source transmits packets to the mobiles (relays) it encounters; relays transmit the packets only if they come in contact with the destination. Thus, each packet travels at most two hops to reach its destination. Multiple variants of the basic two-hop relay [1] has been proposed to support applications of different requirements, like the two-hop relay with multiple copies [2], the two-hop relay with in-order reception [3], and the group-based two-hop relay [4].

Recently, the erasure coding technique has been incorporated into two-hop relay to improve its delay performance [5]. A simple theoretical model was further developed by Hanbali *et al.* in [6] for performance analysis of two-hop relay with erasure coding. It is notable, however, that the model considered only a very simple scenario, where the network has only one source-destination pair, and the source node has only one single packet to deliver to the destination. Thus, this model is not general enough for performance of general MANET scenarios, where multiple traffic flows (source-destination pairs) may co-exist and a relay node may simultaneously carry packets belonging to multiple flows. Also, the important interference, medium contention and traffic contention issues have not been carefully addressed in above model, so it cannot be adopted for an accurate packet delay

analysis.

In this paper, we develop a general and accurate theoretical framework for the performance modeling of two-hop relay with erasure coding. The main contributions of this paper are summarized as follows.

- In the Section III, we develop a general discrete time Markov chain-based theoretical framework to model the complicated message delivery process of the two-hop relay with erasure coding.
- With the help of the theoretical framework, in the Section IV we analytically derive both the mean value and the variance of the packet delivery delay, where the important medium contention, interference and traffic contention issues are carefully incorporated into the analysis.
- Extensive simulation results are provided in the Section V to validate the new theoretical framework, which indicate that the new models can efficiently capture the behaviors of the mean value and the variance of the packet delivery delay in a MANET with two-hop relay and erasure coding.

## II. SYSTEM MODELS AND TRANSMISSION SCHEDULING

### A. System Models

We consider a time slotted system and assume that the network consists of  $n$  mobile nodes inside a unit square, which is evenly divided into  $m \times m$  cells. The bi-dimensional i.i.d. mobility model [3] is adopted here. At the beginning of each time slot, each node independently and uniformly selects a cell among all  $m^2$  cells and stays in it for the whole time slot [7].

We adopt the protocol model [8] as the interference model with  $\Delta$  as the guarding factor. We further assume a permutation traffic pattern [4], where each node has a locally generated message to deliver to its destination, and also need to receive a message from some other node. Thus, there are in total  $n$  distinct flows.

In this paper, we focus on the two-hop relay protocol with erasure coding [5], [6], [9], [10]. Under such a protocol with replication factor  $\tau$ , a message of size  $M$  at some source node is first split into  $\omega$  blocks (each block of size  $M/\omega$ ), and then erasure coded into  $\omega \cdot \tau$  equal sized frames (or code blocks). Each frame is also of size  $M/\omega$ , and any  $(1+\epsilon) \cdot \omega$  frames can

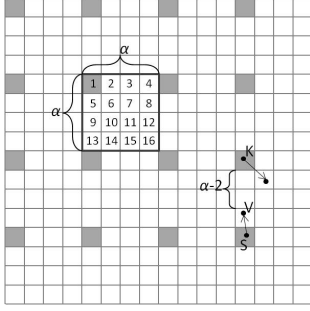


Fig. 1. An example of a concurrent-set of cells with  $\alpha = 4$ . The cells are divided into 16 different concurrent-sets and the shaded cells all belong to the same concurrent-set, i.e., the concurrent-set 1. The distribution of all the remaining nodes in the unit square is not shown for simplicity.

be used to reconstruct the message, here  $\epsilon$  is a small constant and it varies with the erasure coding algorithm adopted [5]. After erasure coding the message into  $\omega \cdot \tau$  frames, each source node starts to deliver these frames according to the two-hop relay protocol [3]. To simplify the analysis, we ignore constant  $\epsilon$  here and thus the message can be successfully recovered at its destination with no less than  $\omega$  frames collected. We further assume that the number of bits that can be successfully transmitted during one time slot is the same as the size of one frame, i.e.,  $M/\omega$ .

### B. Transmission Scheduling

We consider a local transmission scenario where a node can only transmit to other nodes inside the same cell or the eight surrounding adjacent cells (two cells are called adjacent if they share a common point). Thus, the maximum distance between a transmitting node (transmitter) and a receiving node (receiver) is  $\sqrt{8}/m$ , so we set the communication range as  $r = \sqrt{8}/m$ . Similar to the “equivalence class” in the [11], we define here the “concurrent-set”.

**Concurrent-set:** As illustrated by the shaded cells in Fig. 1, a concurrent-set is a subset of cells in which any two cells have a vertical and horizontal distance of some multiple of  $\alpha$  cells, and all the cells there can transmit simultaneously without interfering each other.

As shown in the Fig. 1, suppose that during some time slot, the node  $V$  is scheduled to receive a frame from the node  $S$ . It’s easy to see that except the node  $S$ , another transmitting node (say node  $K$ ) in the same concurrent-set is at least  $(\alpha - 2)/m$  away from  $V$ . According to the protocol model [8], the condition that  $K$  will not interfere with the reception at  $V$  is that,  $(\alpha - 2)/m \geq (1 + \Delta) \cdot r$ . In light of that  $r = \sqrt{8}/m$  and the  $\alpha$  is an integer and  $\alpha \leq m$ , the  $\alpha$  can be determined by

$$\alpha = \min \{ \lceil (1 + \Delta)\sqrt{8} \rceil + 2, m \} \quad (1)$$

where  $\lceil x \rceil$  returns the smallest integer not less than  $x$ .

The concurrent-sets are assumed to become active alternatively, and thus each cell will become active (i.e., get transmission opportunity) in every  $\alpha^2$  time slots. If there are more than one nodes inside an active cell, a transmitting node is selected randomly from them.

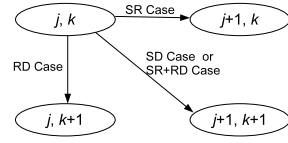


Fig. 2. The transition diagram of the state  $(j, k)$ , where  $1 \leq j < \omega \cdot \tau$ ,  $0 \leq k < \omega$ . The transition back to itself is not shown for simplicity.

Every time a node is selected as the transmitting node, it conducts the Source-to-Destination transmission for its own traffic if its destination node is inside the one-hop neighbor, otherwise, it randomly conducts the Source-to-Relay transmission or the Relay-to-Destination transmission with equal probability [3]. A source node delivers each frame to at most one relay node, and a relay node can carry at most one frame of a particular message.

## III. A MARKOV CHAIN-BASED THEORETICAL FRAMEWORK

### A. Markov Chain Framework

Without loss of generality, we focus on a specific flow. The source node has in total  $\omega \cdot \tau$  frames to deliver, and its destination can decode the original message as long as no less than  $\omega$  frames have been received. Thus, we can denote by state  $(j, k)$  a general transient state among the delivery process, where the source is delivering the  $j_{th}$  frame and its destination has received  $k$  distinct frames.

As indicated in the Fig. 2, for each transient state  $(j, k)$ , it may transit into different neighboring states in the next time slot. The transitions among neighboring states depend on the transmission cases in the current time slot, which are defined as follows:

- (SR case) Source-to-Relay transmission only, i.e., the source successfully delivers the  $j_{th}$  frame to a new relay while none of the relays deliver a frame to the destination;
- (RD case) Relay-to-Destination transmission only, i.e., some relay node successfully delivers a frame to the destination while the source fails to deliver out the  $j_{th}$  frame;
- (SR+RD case) both Source-to-Relay and Relay-to-Destination transmissions, i.e., these two transmissions happen simultaneously;
- (SD case) Source-to-Destination transmission, i.e., the source successfully delivers out a frame to the destination.

Based on the transition diagrams of each state  $(j, k)$ , the message delivery process can be modeled as a discrete-time finite-state absorbing Markov chain. We illustrate the transition diagram of the corresponding absorbing Markov chain in the Fig.3, where state  $(*, k)$  denotes the state that the source has delivered out all the  $\omega \cdot \tau$  frames while the destination has only received  $k$  of them, the  $(a, t)$  denotes the state that the Markov chain gets absorbed by state  $(\omega + t - 1, \omega - 1)$ ,  $t \in [1, \omega \cdot \tau - \omega + 1]$ , and  $(a, \omega \cdot \tau - \omega + 2)$  denotes the state that the Markov chain gets absorbed by state  $(*, \omega - 1)$  (here  $a$  denotes absorbing).

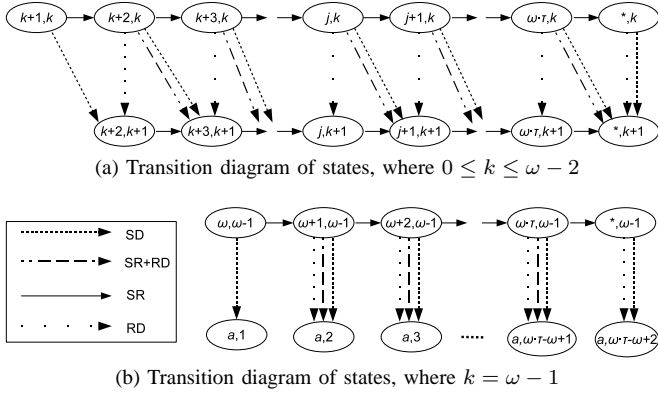


Fig. 3. Transition diagram of the Markov chain for the message delivery process. For each transient state, the transition back to itself is not shown for simplicity.

### B. Related Basic Results

As indicated in the Fig.3, there are  $\omega$  rows of transient states, with  $L_k$  transient states in the  $k_{th}$  row,  $0 \leq k \leq \omega - 1$ , where

$$L_k = \omega \cdot \tau - k + 1 \quad (2)$$

Thus, the Markov chain has in total  $\beta$  transient states where

$$\beta = \frac{\omega}{2}(2\omega \cdot \tau - \omega + 3) \quad (3)$$

For the  $t_{th}$  transient state in the  $k_{th}$  row,  $k \in [0, \omega - 1]$ , we denote by  $u_r$  and  $u_o$  the number of relays carrying frames and the number of relays without frames, respectively, then we can easily see that

$$u_r = t - 1 \quad (4)$$

$$u_o = n - t - 1 \quad (5)$$

We include here some basic probability results related to the Markov model in Fig.3, which will help us to derive the transition probabilities among neighboring transient states. Notice that the important medium contention, interference and traffic contention issues are carefully incorporated into the derivations of the following transition probabilities.

*Lemma 1:* For a time slot and one given source node, we use  $p_1$  and  $p_2$  to denote the probability that the source conducts a Source-to-Destination transmission and the probability that the source conducts a Source-to-Relay or Relay-to-Destination transmission, respectively. Then we have

$$p_1 = \frac{1}{\alpha^2} \left( \frac{9n - m^2}{n(n-1)} - \left(1 - \frac{1}{m^2}\right)^{n-1} \frac{8n + 1 - m^2}{n(n-1)} \right) \quad (6)$$

$$p_2 = \frac{1}{\alpha^2} \left( \frac{m^2 - 9}{n-1} \left(1 - \left(1 - \frac{1}{m^2}\right)^{n-1}\right) - \left(1 - \frac{9}{m^2}\right)^{n-1} \right) \quad (7)$$

*Lemma 2:* For a time slot and one given source node, given that there are  $t_1$  relay nodes carrying frames from the source and  $t_2$  relay nodes without frames, we use  $P_r(t_1)$ ,  $P_d(t_2)$  and  $P_s(t_1, t_2)$  to denote the probability that the destination node will receive a frame, the probability that the source will

successfully deliver out a frame and the probability of simultaneous Source-to-Relay and Relay-to-Destination transmissions in the next time slot, respectively. Then we have

$$P_r(t_1) = p_1 + \frac{t_1}{2(n-2)} p_2 \quad (8)$$

$$P_d(t_2) = \frac{t_2}{2(n-2)} p_2 \quad (9)$$

$$P_s(t_1, t_2) = \frac{t_1 t_2 (m^2 - \alpha^2)}{4m^2 \alpha^4} \sum_{k=0}^{n-5} \binom{n-5}{k} h(k) \left\{ \sum_{t=0}^{n-4-k} \binom{n-4-k}{t} h(t) \left(1 - \frac{18}{m^2}\right)^{n-4-k-t} \right\} \quad (10)$$

where

$$h(x) = \frac{9\left(\frac{9}{m^2}\right)^{x+1} - 8\left(\frac{8}{m^2}\right)^{x+1}}{(x+1)(x+2)} \quad (11)$$

The derivations of the (6), (7), (8), (9) and (10) are omitted here due to the space limit. Please refer to [12] for details.

### IV. EXPECTED VALUE AND VARIANCE OF MESSAGE DELIVERY DELAY

For a specific source-destination pair, the message delivery delay is defined as the time elapsed from the time slot when the source node starts to deliver the first frame, to the time slot when the destination node is able to recover the original message (i.e., when no less than  $\omega$  frames are collected at the destination). We denote this variable by  $T(\tau, \omega)$ , and denote by  $\mathbb{E}\{T(\tau, \omega)\}$  and  $Var\{T(\tau, \omega)\}$  its expected value and variance, respectively.

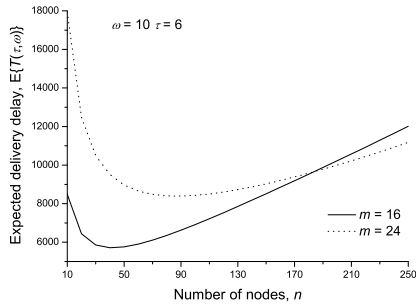
#### A. Derivations of $\mathbb{E}\{T(\tau, \omega)\}$ and $Var\{T(\tau, \omega)\}$

We denote by  $\mathbf{N}$  the fundamental matrix of the Markov chain in the Fig. 3, and further divide the matrix  $\mathbf{N}$  into  $\omega$ -by- $\omega$  blocks with blocks in the  $k_{th}$  ( $1 \leq k \leq \omega$ ) row corresponding to transient states in the  $(k-1)_{th}$  row of the Markov chain. Based on this block partition, the  $ij$ -entry  $\mathbf{N}_{tk}(i, j)$  of the  $tk$ -block  $\mathbf{N}_{tk}$  can be regarded as the expected number of times in the  $j_{th}$  transient state of the  $(k-1)_{th}$  row given that the chain starts from the  $i_{th}$  transient state of the  $(t-1)_{th}$  row. Thus, the  $\mathbb{E}\{T(\tau, \omega)\}$  can be determined as

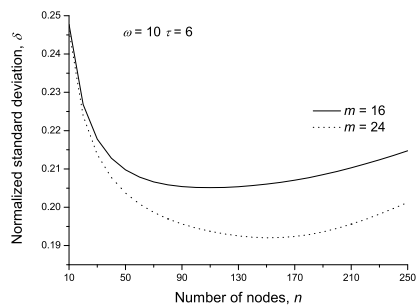
$$\mathbb{E}\{T(\tau, \omega)\} = \sum_{k=1}^{\omega} \sum_{j=1}^{L_{k-1}} \mathbf{N}_{1k}(1, j) \quad (12)$$

As there are  $\omega$  rows of transient states in the Markov chain, we index these transient states sequentially as  $1, 2, \dots, \beta$ , first from left to right then from top to down. Let random variable  $b_i$  denote the time the chain takes to become absorbed given that the chain starts from the  $i_{th}$  transient state ( $1 \leq i \leq \beta$ ). Let  $\mathbf{b}^{(j)} = (\mathbb{E}\{b_1^j\}, \mathbb{E}\{b_2^j\}, \dots, \mathbb{E}\{b_\beta^j\})^T$ , where  $\mathbb{E}\{b_i^j\}$  denotes the  $j_{th}$  raw moment for random variable  $b_i$ ,  $1 \leq i \leq \beta$ , and let  $\mathbf{c}$  be the  $\beta \times 1$  column vector with all of whose entries are 1, then we have





(a) The mean value  $\mathbb{E}\{T(\tau, \omega)\}$  vs. number of nodes  $n$



(b) The normalized standard deviation  $\delta$  vs. number of nodes  $n$

Fig. 5. Message delivery delay vs. number of nodes  $n$ .

where the  $\delta$  denotes the normalized standard deviation,  $\delta = \frac{\sqrt{\text{Var}\{T(\tau, \omega)\}}}{\mathbb{E}\{T(\tau, \omega)\}}$ .

The Fig. 4 shows clearly that the simulation results match nicely with the theoretical ones for both the  $\mathbb{E}\{T(\tau, \omega)\}$  and the  $\delta$ , so our framework can be used to efficiently and accurately model the complicated message delivery process of two-hop relay with erasure-coding. A further careful observation of the Fig. 4a and 4b indicates that the delay performance ( $\mathbb{E}\{T(\tau, \omega)\}$ ,  $\delta$ ) cannot be improved anymore as the  $\tau$  increases beyond a threshold value. For example, when  $m = 16$ ,  $n = 100$ ,  $\omega = 8$ , the  $\mathbb{E}\{T(\tau, \omega)\}$  (resp. the  $\delta$ ) first decreases from 12136 (resp. 0.35) down to 5959 (resp. 0.232) as the  $\tau$  increases from 1 up to 5, then remains 5953 (resp. 0.23) as the  $\tau$  further increases from 6 to 12. Regarding the case that  $m = 8$ ,  $n = 50$ ,  $\omega = 6$ , the  $\mathbb{E}\{T(\tau, \omega)\}$  (resp. the  $\delta$ ) remains 2897 (resp. 0.331) as the  $\tau$  increases from 4 to 12. Thus, we can see that for a given network scenario of  $m$ ,  $n$  and  $\omega$ , there exists a corresponding threshold for  $\tau$ , beyond which a further increase of  $\tau$  will not introduce additional performance improvement for both  $T(\tau, \omega)$  and  $\delta$ .

Based on the above framework, we further proceed to explore in Fig. 5 how the message delivery delay varies with the number of nodes  $n$  for the scenarios of  $m = \{16, 24\}$ . We can see from the Fig. 5a that there exists an optimum value of  $n$  which minimizes the  $\mathbb{E}\{T(\tau, \omega)\}$ . For example, when  $m = 16$  (resp.  $m = 24$ ), the minimum  $\mathbb{E}\{T(\tau, \omega)\}$  of 5717

(resp. 8395) is achieved at  $n = 40$  (resp.  $n = 90$ ). Similarly, the Fig. 5b shows that for the cases  $m = 16$  and  $m = 24$ , the minimum  $\delta$  of 0.205 and 0.192 are achieved at  $n = 110$  and  $n = 150$ , respectively. The Fig. 5a also indicates that the  $n$  and thus the average node density  $n/m^2$  can significantly affect the  $\mathbb{E}\{T(\tau, \omega)\}$ . For example, when  $m = 16$ , the  $\mathbb{E}\{T(\tau, \omega)\}$  of the  $n = 250$  is 12013, which is nearly 1.74 times as that of the  $n = 100$  (6914 there).

## VI. CONCLUSION

This paper provided the performance modeling for the two-hop relay protocol with erasure coding. In order to model the complicated message delivery process in the MANETs, a discrete time Markov chain-based theoretical framework was developed, based on which not only the mean value but also the variance of the message delivery delay were analytically derived. As verified through extensive simulation results, the new model is very efficient for the message delivery delay analysis of two-hop relay protocol with erasure coding. Our results in this paper indicate that for a given network, a threshold for the replication factor can be determined, beyond which further performance improvement for both  $T(\tau, \omega)$  and  $\delta$  cannot be achieved.

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