Message Delivery Probability of Two-Hop Relay with Erasure Coding in MANETs

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***Abstract*—The lack of a thorough understanding of the fundamental performance limits in mobile ad hoc networks (MANETs), remains a challenging roadblock stunting the commercialization and application of such networks. In this paper, we consider a MANET employing the two-hop relay algorithm and erasure coding, and focus on the message delivery probability there. Specifically, a finite-state absorbing Markov chain framework is first developed to characterize the complicated message delivery process in the challenging MANETs. Based on the developed framework, closed-form expressions are further derived for the message delivery probability under any given message lifetime and message size by adopting the blocking matrix technique. As verified through extensive simulation studies, the new framework can be used to accurately predict the message delivery probability behavior, and characterize its relationship with the message size, replication factor and node density.**

I. Introduction

A mobile ad hoc network (MANET) is a peer-to-peer network without any pre-existing infrastructure or centralized administration, which consists of fully self-organized mobile nodes. As it can be rapidly deployed and flexibly reconfigured, the MANET has found many promising applications, such as the disaster relief, emergency response, daily information exchange, etc., and thus becomes an indispensable component among the next generation networks [1].

By now, a significant amount of work has been done for a thorough understanding of the fundamental performance limits in MANETs. It was proved that by employing a two-hop relay algorithm (or its variant), a $\Theta(1)$ per node throughput can be achieved under the i.i.d. mobility model [2], the random walk model [3] and other mobility models [4]. The scaling laws of packet delay in MANETs and its trade-off with the throughput capacity have also been intensively addressed in literature, like [5], [6]. Aside from these order sense results, more recently, some closed-form results have also been reported for the packet delivery delay [7], the end-to-end packet delay [8] and the exact achievable throughput capacity [9]. In this paper, we focus on the performance of message delivery probability in MANETs.

It is noticed that there is some prior work addressing the message delivery probability in literature. Panagakis *et al.* in [10] analytically derived the message delivery probability of the two-hop relay under a given time limit by approximating the CDF of message delivery delay, where it is assumed that for any node pair, the message can be successfully transmitted whenever they meet each other. Whitbeck *et al.* in [11] explored the impact of message size, message lifetime and link lifetime on the message delivery ratio (probability) of epidemic routing by treating the intermittently connected mobile networks as edge-Markovian graphs, where each link (edge) is considered independent and has the same transition probabilities between "up" and "down" status.

Obviously, the available models in [10], [11] considered too simple network scenarios and cannot be applied for the general MANETs, where the interference and medium contention issues are of significant importance and thus the network cannot be simplified as edge-Markovian graphs. Furthermore, since the number of data bits that can be successfully transmitted during a node contact is actually limited and the buffer space allocated at each mobile node may also be very limited, a message may need to be split into multiple blocks such that each block can be successfully transmitted during a contact or stored at the relay buffer. In this paper, we develop closed-form models for the message delivery probability in MANETs with a careful consideration of the above important issues. The main contributions of this paper are summarized as follows.

- We focus on the two-hop relay with erasure coding, where a message at the source node is first erasure coded into multiple frames (coded blocks) before transmitting. In Section III, we develop a finite-state absorbing Markov chain framework to model the complicated message spreading process in the challenging MANETs.
- Based on the theoretical framework, we further derive closed-form expressions in Section III for the corresponding message delivery probability under any given message lifetime and message size, where the important issues of interference, medium contention and traffic contention in MANETs are carefully incorporated into the analysis.
- In Section IV, extensive simulation studies are conducted to validate our theoretical framework, which indicate that the new framework can be used to accurately predict the message delivery probability in MANETs with two-hop relay and erasure coding, and characterize how the parameters of message size, replication factor and node density would affect the delivery probability there.

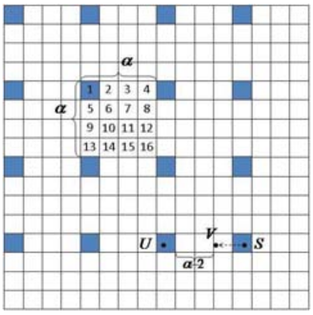


Fig. 1. Illustration of cell-partitioned network and cells in a transmission-group with $m = 16$ and $\alpha = 4$.

II. Preliminaries

A. System Models

The considered mobile ad hoc network is a unit torus with n mobile nodes. The torus is evenly divided into $m \times m$ equal cells (or squares), each cell of side length $1/m$ as shown in Fig. 1. Time is slotted, and nodes randomly roam from cell to cell according to the i.i.d. mobility model [6]. At the beginning of each time slot, each node randomly selects a destination cell among the m^2 cells, and then move inside and stay for a whole time slot. Similar to [5], we assume that a whole time slot will be allocated only for data transmissions in one hop range. Each node employs a common transmission range r, and the protocol model with guard factor Δ (a positive constant representing the guard zone) [12] is adopted to address the interference among simultaneous transmitting links.

In order to fully characterize the traffic contention issue in MANETs, we consider here the permutation traffic pattern [7], where each node has a locally generated message to deliver to its destination node and also needs to receive another message originated from some other node. It is easy to see that there are in total n distinct traffic flows. Without loss of generality, we focus on a tagged flow hereafter and denote its source and destination by S and D, respectively.

For the tagged flow, the message generated at the source S is assumed to have in total ω blocks ($\omega \geq 1$), where a single block can be successfully transmitted during a time slot (or meeting duration). We further assume that the message is relevant during τ time slots, i.e., the message is labeled with a lifetime of τ time slots after it is generated at S, and will be dropped from the network if it fails to make itself to the destination D within τ time slots.

B. Two-Hop Relay with Erasure Coding

According to the two-hop relay algorithm with erasure coding [13], [14], for the tagged flow, the message is first erasure coded into $\omega \cdot \beta$ equal sized frames (or code blocks) after it is locally generated at S, where β is the replication factor. Since each frame is almost the same size as the original block, it can also be successfully transmitted during a time slot. Any $(1+\epsilon)\cdot\omega$ frames can be used to successfully reconstruct the message, where ϵ is a small constant and it varies with the adopted erasure coding algorithm. Similar to [13], [14], we ignore the constant ϵ here and thus the message can be successfully recovered at the destination D with no less than ω frames collected before it expires (or within τ time slots).

After erasure coding the message into $\omega \cdot \beta$ frames, the source node S starts to deliver out these frames according to the two-hop relay algorithm [2], [5], [6]. Every time S wins a transmitting opportunity, it operates as follows:

Step 1: S first checks whether D is in the transmission range. If so, S conducts with D the "source-to-destination" transmission, where a frame (if not expired) is sent to D.

Step 2: Otherwise, S randomly selects a node, say R, from the one-hop neighborhood as the receiver, and then conducts with R either the "source-to-relay" transmission or the "relay-to-destination" transmission with equal probability. In the "source-to-relay" transmission, S acts as a source and sends to R a frame (if not expired) destined for D; while in the "relay-to-destination" transmission, S acts as a relay and forwards to R a frame (if available) destined for R.

Notice that in the frame distribution process at node S, each frame will be delivered to at most one relay node, and each relay node will also carry at most one frame from S.

C. Transmission Scheduling

Similar to previous studies [5], [7], [15], we consider a local transmission scenario where a transmitter in some cell can only transmit to receivers in the same cell or other eight adjacent cells (two cells are called adjacent cells if they share a common point). Thus, the transmission range can be accordingly determined as $r = \sqrt{8}/m$. It is easy to see that two links can transmit simultaneously if and only if they are sufficiently far away from each other. To avoid collisions among simultaneous transmitting links and support as many simultaneous link transmissions as possible, we adopt here the transmission-group based scheduling scheme [5], [15].

Transmission-group: A transmission-group is a subset of cells where any two of them have a vertical and horizontal distance of some multiple of α cells and all the cells there could transmit simultaneously without interfering with each other.

With such a transmission-group definition, all m^2 cells are actually divided into α^2 distinct transmission-groups. If each transmission-group becomes active (i.e., has link transmissions) alternatively, then each cell will also become active every α^2 time slots. As illustrated in Fig. 1 for the case $\alpha = 4$, there are in total 16 transmission-groups, and all shaded cells belong to the same transmission-group.

Setting of Parameter α: As shown in Fig. 1, suppose node S in an active cell is transmitting to node V in some time slot. It is easy to see that another transmitter, say U, in some other active cell is at least $\alpha-2$ cells away from V. According to the protocol interference model [12], we should have $(\alpha-2)\cdot\frac{1}{m} \geq (1+\Delta)\cdot r$ to ensure the successful data reception at V. Notice that $\alpha \leq m$ and $r = \sqrt{8}/m$, then the parameter α can be determined as

$$\alpha = \min\{\lceil (1+\Delta)\sqrt{8} + 2\rceil, m\} \qquad (1)$$

III. Message Delivery Probability

A. Some Basic Probabilities

Before proceeding to introduce the Markov chain theoretical framework and derive the expected message delivery probability, we first present here some basic probabilities under the two-hop relay with erasure coding.

Lemma 1: For a time slot and the tagged flow, if we denote by p_1 the probability that S conducts a "source-to-destination" transmission with the destination node D and denote by p_2 the probability that S conducts a "source-to-relay" transmission or "relay-to-destination" transmission with some other node, then we have

$$p_1 = \frac{1}{\alpha^2}\left(\frac{9n-m^2}{n(n-1)} - \left(1-\frac{1}{m^2}\right)^{n-1}\frac{8n+1-m^2}{n(n-1)}\right) \quad (2)$$

$$p_2 = \frac{1}{\alpha^2}\left(\frac{m^2-9}{n-1}\left(1-\left(1-\frac{1}{m^2}\right)^{n-1}\right)-\left(1-\frac{9}{m^2}\right)^{n-1}\right) \quad (3)$$

Lemma 2: For a time slot and the tagged flow, given that there are t_1 relay nodes each carrying a frame from the source node S and t_2 relay nodes carrying no frames from S, we denote by $p_r(t_1)$, $p_d(t_2)$ and $p_s(t_1,t_2)$ the probability that the destination node D will receive a frame, the probability that S will successfully deliver out a frame to some new relay node (if $t_1 < \omega\cdot\beta$), and the probability of simultaneous "relay-to-destination" transmission (where D obtains a frame from the t_1 relay nodes) and "source-to-relay" transmission (where S delivers out a frame to the t_2 relay nodes) in the next time slot. Then we have

$$p_r(t_1) = p_1 + \frac{t_1}{2(n-2)}p_2 \quad (4)$$

$$p_d(t_2) = \frac{t_2}{2(n-2)}p_2 \quad (5)$$

$$p_s(t_1,t_2) = \frac{t_1 t_2 (m^2-\alpha^2)}{4m^2\alpha^4}\sum_{k=0}^{n-5}\binom{n-5}{k}h(k) \cdot\left\{\sum_{t=0}^{n-4-k}\binom{n-4-k}{t}h(t)\left(1-\frac{18}{m^2}\right)^{n-4-k-t}\right\} \quad (6)$$

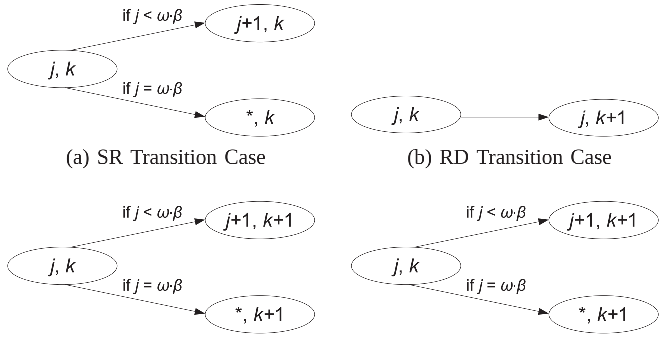
where

$$h(x) = \frac{9\left(\frac{9}{m^2}\right)^{x+1} - 8\left(\frac{8}{m^2}\right)^{x+1}}{(x+1)(x+2)} \quad (7)$$

The derivations of (2), (3), (4), (5) and (6) are omitted here due to space limit, and please refer to [7] for details.

B. Markov Chain Framework

For the tagged flow, as the message generated at the source node S is erasure coded into $\omega\cdot\beta$ frames and is relevant only in τ time slots, the destination node D needs to collect at least ω frames within τ time slots so as to successfully recover the message. If we denote by (j,k) a general transient state during the message delivery process that S is delivering the j_{th} frame and D has already received k distinct frames, and further denote by $(*,k)$ a transient state that S has already finished dispatching all $\omega\cdot\beta$ frames while D has only received k of them, $1\leq j\leq\omega\cdot\beta$, $0\leq k<\omega$, then we can characterize the message delivery process with a finite-state absorbing Markov chain. Specifically, if the tagged flow is in state (j,k) at the current time slot, only one of the following four transition cases illustrated in Fig. 2 may happen in the next time slot.



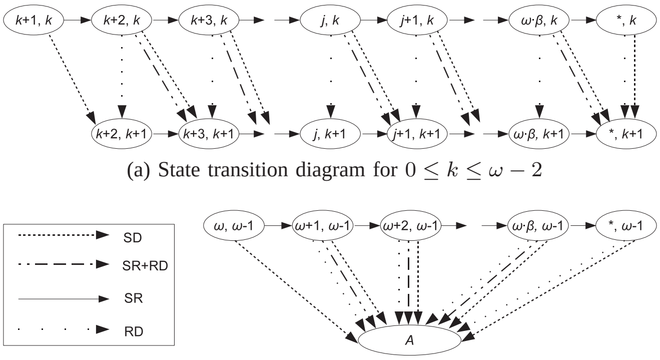
(a) SR Transition Case (b) RD Transition Case

(c) SR+RD Transition Case (d) SD Transition Case

Fig. 2. Transition cases of a general transient state (j,k), where $1\leq j\leq \omega\cdot\beta$, $0\leq k<\omega$, $k<j$.

- SR Case: "source-to-relay" transmission only, i.e., S successfully delivers the j_{th} frame to a new relay node while none of the relays delivers a frame to D. As shown in Fig. 2a that under such a transition case, the state (j,k) may transit to two different neighboring states depending on the current frame index j.
- RD Case: "relay-to-destination" transmission only, i.e., some rely node successfully delivers a frame to D while S fails to deliver out the j_{th} frame to a new relay node. As shown in Fig. 2b that there is only one target state $(j,k+1)$ under the RD case.
- SR+RD Case: both "source-to-relay" and "relay-to-destination" transmissions, i.e., these two transmissions happen simultaneously. We can see from Fig. 2c that depending on the value of j there are two possible target states under the SR+RD case.
- SD Case: "source-to-destination" transmission only, i.e., S successfully delivers a frame to D. As shown in Fig. 2d that under the SD case, the state (j,k) may transit to $(j+1,k+1)$ or $(*,k+1)$, similar to that under the SR+RD case.

If we denote by A the absorbing state that the destination node D has collected ω distinct frames, then the transition diagrams in Fig. 2 indicate that the message delivery process can be modeled as a discrete-time finite-state absorbing Markov chain illustrated in Fig. 3, where Figs. 3a and 3b each represents some cases of the full chain. Specifically, Fig. 3a represents the cases that D may receive at most one more frame given that it has already received k frames, $0\leq k\leq\omega-2$; Fig. 3b shows the transition diagrams of how D may receive the last frame. The transitions of



(a) State transition diagram for $0 \leq k \leq \omega - 2$

(b) State transition diagram for $k = \omega - 1$

Fig. 3. Transition diagram of the Markov chain for the message delivery process. For each transient state, the transition back to itself is not shown for simplicity.

SR, RD, SD and SR+RD in Fig. 3 correspond to two-hop transmissions of "source-to-relay", "relay-to-destination", "source-to-destination" and both "source-to-relay" and "relay-to-destination", respectively.

As shown in Fig. 3, there are in total ω rows of transient states, with L_k transient states in the k_{th} $(0 \leq k \leq \omega - 1)$ row, where

$$L_k = \omega \cdot \beta - k + 1 \tag{8}$$

Therefore, the total number of transient states δ in Fig. 3 can be determined as

$$\delta = \frac{\omega}{2}(2\omega \cdot \beta - \omega + 3) \tag{9}$$

Consider the t_{th} transient state of the k_{th} row in the Markov chain of Fig. 3, $0 \leq k \leq \omega - 1$, $1 \leq t \leq L_k$, if we denote by u_r the number of relay nodes each carrying a frame from node S, and denote by u_o the number of relay nodes carrying no frames from S, then we have

$$u_r = t - 1 \tag{10}$$

$$u_o = n - t - 1 \tag{11}$$

C. Derivations of Delivery Probability $\varphi(\omega, \beta, \tau)$

Before deriving the message delivery probability, we first introduce the following definition.

Definition 1: For a message locally generated at the source node S which is further erasure coded into $\omega \cdot \beta$ frames, the delivery delay of the message is defined as the time elapsed between the time slot when S starts to deliver the first frame of the message and the time slot when the destination node D receives the ω_{th} frame (i.e., when the destination D is able to recover the message).

For the tagged flow, if we denote by T_d the message delivery delay and denote by $\varphi(\omega, \beta, \tau)$ the message delivery probability under the message lifetime constraint τ, then we have

$$\varphi(\omega, \beta, \tau) = \Pr(T_d \leq \tau) = \sum_{t=1}^{\tau} \Pr(T_d = t) \tag{12}$$

Based on the Markov chain framework, now we are ready to derive $\varphi(\omega, \beta, \tau)$. As shown in Fig. 3, all δ transient states in the Markov chain are arranged into ω rows. We number these transient states sequentially as $1, 2, \ldots, \delta$ in a left-to-right and top-to-down way. For these transient states, if we let q_{ij} denote the transition probability from state i to state j, then we can define a matrix $\mathbf{Q} = (q_{ij})_{\delta \times \delta}$ of transition probabilities among δ transient states there. Similarly, if we let b_i denote the one-step transition probability from state i to the absorbing state A, then we can also define a vector $\mathbf{B} = (b_i)_{\delta \times 1}$ representing the transition probabilities from δ transient states to state A.

Notice that $\Pr(T_d = t)$ in (12) denotes the probability that the ω_{th} frame arrives at the destination D by the end of the t_{th} time slot, i.e., the probability that the Markov chain gets absorbed by the end of the t_{th} time slot. Given that the Markov chain starts from the first state, i.e., state $(1, 0)$, according to the Markov chain theory [16], then we have

$$\Pr(T_d = t) = \sum_{i=1}^{\delta} q_{1i}^{(t-1)} \cdot b_i \tag{13}$$

where $q_{ij}^{(t)}$ denotes the probability that by the end of the t_{th} time slot the Markov chain is in the j_{th} state given that the Markov chain starts from the i_{th} state.

Combining with the fact that $q_{ij}^{(t)}$ is actually the ij-entry of the matrix $\mathbf{Q}^t$, i.e., $\mathbf{Q}^t = (q_{ij}^{(t)})_{\delta \times \delta}$, (13) can be further transformed as

$$\Pr(T_d = t) = \mathbf{e} \cdot \mathbf{Q}^{t-1} \cdot \mathbf{B} \tag{14}$$

where $\mathbf{e} = \{1, 0, \ldots, 0\}$.

Substituting (14) into (12), then we have

$$\begin{aligned} \varphi(\omega, \beta, \tau) &= \sum_{t=1}^{\tau} \mathbf{e} \cdot \mathbf{Q}^{t-1} \cdot \mathbf{B} \\ &= \mathbf{e} \cdot (\mathbf{I} - \mathbf{Q})^{-1} \cdot (\mathbf{I} - \mathbf{Q}^{\tau}) \cdot \mathbf{B} \\ &= \mathbf{e} \cdot \mathbf{N} \cdot (\mathbf{I} - \mathbf{Q}^{\tau}) \cdot \mathbf{B} \end{aligned} \tag{15}$$

where $\mathbf{I}$ is the identity matrix, and $\mathbf{N} = (\mathbf{I} - \mathbf{Q})^{-1}$ is the fundamental matrix of the Markov chain in Fig. 3.

From (15) we can see that in order to derive the message delivery probability $\varphi(\omega, \beta, \tau)$, the only remaining issue is to derive the matrices $\mathbf{Q}$, $\mathbf{N}$ and $\mathbf{B}$, as introduced in the following section.

D. Derivations of Matrices $\mathbf{Q}$, $\mathbf{N}$ and $\mathbf{B}$

Notice that for the Markov chain in Fig. 3, the transitions happen only among transient states of the same row or neighboring rows, and thus the matrix $\mathbf{Q}$ can be defined as

$$\mathbf{Q} = \begin{bmatrix} \mathbf{Q}_0 & \mathbf{Q}_0^{'} & & & & & \\ & \mathbf{Q}_1 & \mathbf{Q}_1^{'} & & & & \\ & & \ddots & \ddots & & & \\ & & & \mathbf{Q}_k & \mathbf{Q}_k^{'} & & \\ & & & & \ddots & \ddots & \\ & & & & & \mathbf{Q}_{\omega-2} & \mathbf{Q}_{\omega-2}^{'} \\ & & & & & & \mathbf{Q}_{\omega-1} \end{bmatrix} \tag{16}$$

where the block (or sub-matrix) $\mathbf{Q}_k$ of size $L_k \times L_k$ defines the probabilities of transitions among the k_{th} row of the Markov chain, $\mathbf{Q}_k^{'}$ of size $L_k \times L_{k+1}$ defines the probabilities of transitions from the k_{th} row to the $(k+1)_{th}$ row, and all other blocks are zero matrices and thus omitted here for simplicity.

Now we proceed to derive the blocks $\{\mathbf{Q}_k\}$ and $\{\mathbf{Q}_k^{'}\}$.

Definitions of $\mathbf{Q}_k$: Let $\mathbf{Q}_k(i,j)$ denote the ij-entry of the block $\mathbf{Q}_k$, $i,j \in [1, L_k]$, then the non-zero entries of $\mathbf{Q}_k$ can be defined as:

$$\mathbf{Q}_k(i,i) = \begin{cases} 1 + p_s(u_r,u_o) - p_r(u_r) - p_d(u_o) \\ \qquad\qquad\qquad \text{if } 1 \le i < L_k \\ 1 - p_r(u_r) \quad \text{if } i = L_k \end{cases} \tag{17}$$

$$\mathbf{Q}_k(i,i+1) = p_d(u_o) - p_s(u_r,u_o) \quad \text{if } 1 \le i < L_k \tag{18}$$

Definitions of $\mathbf{Q}_k^{'}$: The block $\mathbf{Q}_k^{'}$ is of size $L_k \times L_{k+1}$, where its non-zero ij-entry $\mathbf{Q}_k^{'}(i,j)$ is defined as follows.

$$\mathbf{Q}_k^{'}(i,i) = p_1 + p_s(u_r,u_o) \quad \text{if } 1 \le i < L_k \tag{19}$$

$$\mathbf{Q}_k^{'}(i,i-1) = \begin{cases} p_r(u_r) - p_1 - p_s(u_r,u_o) \\ \qquad\qquad \text{if } 2 \le i < L_k \\ p_r(u_r) \quad \text{if } i = L_k \end{cases} \tag{20}$$

Since the fundamental matrix $\mathbf{N} = (\mathbf{I} - \mathbf{Q})^{-1}$, we can derive $\mathbf{N}$ based on the matrix $\mathbf{Q}$. Please refer to [7] for the details of derivation for matrix $\mathbf{N}$.

Now we proceed to define the matrix $\mathbf{B}$. It is easy to see that $\mathbf{B}$ can also be defined as

$$\mathbf{B} = (\mathbf{0}, \mathbf{0}, \ldots, \mathbf{B}_{\omega-1})^T \tag{21}$$

where $\mathbf{0}$ is the zero matrix.

Definitions of $\mathbf{B}_{\omega-1}$: The block $\mathbf{B}_{\omega-1}$ is of size $L_{\omega-1} \times 1$, where its non-zero ij-entry $\mathbf{B}_{\omega-1}(i,j)$ can be defined as:

$$\mathbf{B}_{\omega-1}(i,1) = p_r(u_r) \quad \text{if } 1 \le i \le L_{\omega-1} \tag{22}$$

Combining (15), (16), (17), (18), (19), (20), (21) and (22), then we get matrices $\mathbf{Q}$, $\mathbf{N}$ and $\mathbf{B}$, and thus the message delivery probability $\varphi(\omega,\beta,\tau)$.

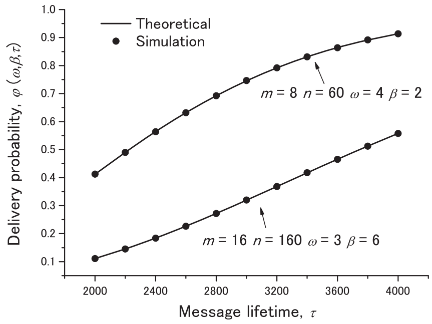


Fig. 4. Comparisons between simulation results and theoretical ones under the network scenarios of $(m = 8, n = 60, \omega = 4, \beta = 2)$ and $(m = 16, n = 160, \omega = 3, \beta = 6)$.

IV. Numerical Results

A. Simulation Settings

A specific simulator was developed to simulate the message delivery process under the two-hop relay algorithm with erasure coding, which is now available at [17]. Similar to the settings in [18], Δ is fixed as $\Delta = 1$ and thus the transmission-group is defined with $\alpha = \min\{8, m\}$. For each network setting of $(m, n, \omega, \beta, \tau)$, the simulated message delivery probability was calculated as the average value of 10^2 batches of simulation results, where each batch consists of 10^4 random and independent simulations.

B. Model Validation

Extensive simulation studies have been conducted to validate the Markov chain theoretical framework developed for the message delivery probability under a given message lifetime τ. Here only two network scenarios, $(m = 8, n = 60, \omega = 4, \beta = 2)$ and $(m = 16, n = 160, \omega = 3, \beta = 6)$, were presented and the results of other scenarios can also be easily obtained by our simulator as well [17]. The corresponding simulation and theoretical results were summarized in Fig. 4. Fig. 4 indicates clearly that for both the network scenarios there, the simulation results match nicely with the theoretical ones, so our framework can be used to efficiently model the message delivery process and accurately characterize the message delivery probability.

C. Performance Analysis

Based on the developed theoretical framework, now we proceed to explore the impact of message size ω on the message delivery probability $\varphi(\omega,\beta,\tau)$. With m and τ fixed as $m = 8$ and $\tau = 3000$, two network settings $(n = 100, \beta = 4)$ and $(n = 45, \beta = 2)$ were examined. One can easily observe from Fig. 5 that, the message delivery probability diminishes quickly as the message size ω increases up. For example, for the setting of $n = 100, \beta = 4$, the message delivery probability at $\omega = 2$ is 0.88, which is almost 5.18 times that of $\omega = 6$ (0.17). A further careful observation of Fig. 5 indicates that the message delivery probability of $n = 100, \beta = 4$ is almost

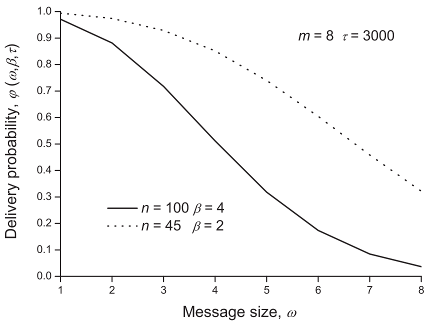


Fig. 5. Impact of message size ω on the message delivery probability under the setting $m = 8, \tau = 3000$.

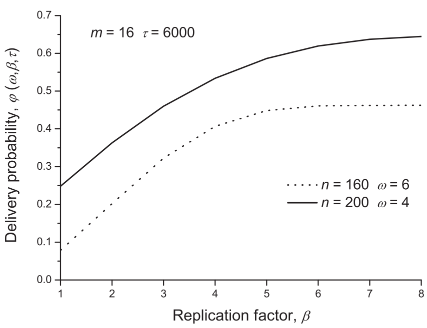


Fig. 6. Impact of replication factor β on the message delivery probability under the setting $m = 16, \tau = 6000$.

0 (0.04) as ω approaches 8, while that of $n = 45, \beta = 2$ remains 0.32 at $\omega = 8$. Combining with Fig. 4 the message lifetime τ, therefore, should be carefully tuned according to the message size ω, replication factor β and node density (i.e., n/m^2) so as to guarantee a specified message delivery.

Fig. 6 illustrates how the replication factor β will affect the message delivery probability $\varphi(\omega, \beta, \tau)$. It is easy to see that for both settings of $(n = 160, \omega = 6)$ and $(n = 200, \omega = 4)$ there, the message delivery probability $\varphi(\omega, \beta, \tau)$ monotonically increases with β. It is interesting to observe from Fig. 6 that, the slope of each curve (i.e., the increasing tendency) decreases as β increases up, and there exists some threshold value of β, beyond which the message delivery probability performance has almost no improvement. Specifically, for the setting $n = 160, \omega = 6$, the message delivery probability remains nearly unchanged as 0.46 when $\beta \geq 6$.

V. Conclusion

In this paper, we have investigated the message delivery probability in MANETs adopting the two-hop relay algorithm with erasure coding. A general Markov chain theoretical framework was developed to characterize the message delivery process, based on which closed-form expressions were derived for the message delivery probability under any given message lifetime and message size. As verified by extensive simulation studies, our framework can be used to efficiently model the message delivery process and thus accurately characterize the message delivery probability there. Our results indicate that the message lifetime parameter τ should be carefully tuned according to the message size ω, replication factor β and node density so as to guarantee a specified delivery performance. Furthermore, in a given MANET there exists some threshold value for the replication factor β, beyond which the message delivery probability cannot be improved any more.

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