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Optimal Rate Selection Scheme in a Two-hop Relay Network Adopting Chase Combining HARQ in Rayleigh Block-fading Channels

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***Abstract*—In Rayleigh block fading channels which represent fast-varying channels, long-term rate adaptation is required instead of instantaneous rate adaptation because the channel information fed back may be outdated. We maximize the long-term average transmission rate (LATR) in a two-hop relay network which adopts Chase combining (CC) type Hybrid Automatic-Repeat-reQuest (HARQ). The *round transmission rate*, i.e. the transmission rate of each HARQ round in each hop, is optimally selected based on the channel statistics of two hops. Two constraints are considered: the outage probability and the maximum number of HARQ rounds, L. In an infinite L case, we show that the optimal *round transmission rate* of one hop is determined only by the channel statistics of that hop, and can be expressed as a Lambert W function. In a finite L case, we propose a numerical search algorithm to find the optimal *round transmission rate*. If HARQ is not adopted, the LATR performance becomes very poor. As L increases in the two-hop relay with CC-based HARQ, the LATR performance becomes close to the LATR performance in the infinite L case. We also show the benefits of the proposed rate selection method compared to a non-optimal rate selection method in terms of the LATR.**

***Index Terms*—HARQ, Multi-hop relay, rate adaptation, Rayleigh block-fading.**

I. Introduction

Hybrid Automatic-Repeat-reQuest (HARQ) schemes have been widely used to achieve temporal diversity and power gains through retransmissions at a transmitter and combining of two multiple received signals at a receiver when earlier transmissions were not successfully decoded [1]–[3]. The HARQ schemes provide robustness against channel fluctuations for reliable communication [4], [5]. When HARQ is operated in a slowly-varying channel, the throughput can be improved by adapting the data rate to the quasi-static channel state [6], [7]. In a fast-varying channel, however, since it is hard for the transmitter to obtain the up-todate instantaneous channel state information, it is rather difficult to use the instantaneous rate adaptation. Instead, long-term rate adaptation can be achieved by using long-term channel statistics. Wu and Jindal [5] proposed a rate adaptation scheme in order to maximize the average transmission rate while satisfying a target outage probability for a single-hop HARQ transmission in Rayleigh block-fading channels. In this paper, extending this rate adaptation scheme, we investigate the optimal rate selection of a two-hop relay adopting a CC-based HARQ scheme in Rayleigh block-fading channels.

A multi-hop relay network generally improves throughput and reliability in wireless networks by taking advantages of coverage extension [8]. Zhao and Valenti [9] presented a practical relay network adopting a generalized HARQ scheme[1] and analyzed the throughput assuming a fixed (not adapted) data rate in Rayleigh block-fading channels. Several recent studies have dealt with cooperative relay protocols exploiting HARQ in a three-node relay network [10]–[13]. In cooperative relay protocols, the signal from the source node reaches the destination so that the destination node obtains a diversity gain exploiting two communication paths from the source and relay nodes. In many practical relay systems, a direct link between the source and destination is not considered due to a complicated control mechanism. Recent standards also have considered multi-hop relay networks [14], [15].

In this paper, we optimize the HARQ operation in a two-hop relay network. We consider two constraints: 1) the maximum number of HARQ rounds, in other words, the maximum number of retransmissions including the first transmission and 2) the information theoretic outage probability. The packet transmission failure probability that a packet is not successfully decoded at the destination even after the maximum number of HARQ rounds, should be lower than a given constrained outage probability. In addition, we let the utility function be the long-term average transmission rate (LATR) which is defined as the average number of information bits attempted to be transmitted per channel use. We consider a Chase combining (CC)-type HARQ scheme [1] which simply combines the retransmitted signal and the previously transmitted signals for the same packet, following the maximum ratio combining (MRC) rule at the receiver. The LATR performance depends on the *round transmission rate*[2] in relay networks. In a two-hop relay network, the optimal *round transmission rates* for those two hops may be different since the two hops have different statistics of channel gains. Therefore, we need to optimize the *round transmission rate* for each hop. That optimization is targeted at maximizing LATR and it is

[1] In the generalized HARQ scheme, the retransmitted packets do not need to come from the original source but could be sent by relays that overhear the transmission.

[2] The *round transmission rate* is the transmission rate of the each HARQ round and it is constant for every HARQ round in a single hop if CC-type HARQ scheme is adopted.

constrained by maintaining both a given maximum number of HARQ rounds and a given outage probability. We first obtain the optimal round transmission rates in a two-hop relay without HARQ schemes and then we consider the optimization problem for a two-hop relay with HARQ which is also either constrained by a delay limit or not. The delay constraint affects the maximum number of HARQ rounds. In the case without a delay constraint, the optimal *round transmission rates* can be expressed as a well-known Lambert W function. We do not require any numerical search in this case. Moreover, we show that the *round transmission rate* in one hop is independent of the channel statistics in another hop. Several schemes have been proposed to find the optimal modulation and coding scheme (MCS) levels in two-hop relay networks [16], [17]. However, we consider both the information-theoretic outage probability and long-term average transmission rate so that our results provide a theoretical performance bound in Rayleigh-block fading channel. In summary, the main contribution of this paper is to optimize the *round transmission rate* of each hop in order to maximize the LATR of a two-hop relay network with CC-type HARQ in Rayleigh block-fading channels.

The rest of this paper is organized as follows: In Section II, we describe a system model in a two-hop relay network. In Section III, we analyze both the outage probabilities and LATRs of the two-hop relay with/without HARQ. Moreover, we formulate optimization problems and propose how to derive the optimal *round transmission rates*. In Section IV, we compare the LATR performance of the two-hop relay schemes. Finally, we present conclusive remarks in Section V.

II. System Model

We consider a two-hop relay network with three nodes: a source node (S), a relay node (R), and a destination node (D) where the relay forwards a packet transmitted by the source to the destination. We assume that there is no direct link between the source and the destination. We assume a block-fading channel where the channel gain is constant during one transmission block, but the channel gains of different blocks are independent and identically distributed (i.i.d.). Let h_{AB} denote the channel coefficient between nodes A and B. It is modeled as an independent, zero-mean complex Gaussian random variable with variance σ^2_{AB}. Let P_{A} denote the transmit power of node A. We assume that each packet has b information bits and T_1 and T_2 symbols are consumed for transmitting the packet in the SR-link and the RD-link, respectively. Then, the transmission rate is $R_1 \triangleq \frac{b}{T_1}$ and $R_2 \triangleq \frac{b}{T_2}$ (bits/symbol or bps/Hz) for the SR-link and the RD-link, respectively. For HARQ schemes, the transmission rate of a packet depends on the number of HARQ rounds used for the packet transmission. For a single link, we let N_i denote the number of HARQ rounds used for the i-th packet. The average transmission rate over M packets becomes $\frac{Mb}{T\sum_{i=1}^{M} N_i} = \frac{R}{\frac{1}{M}\sum_{i=1}^{M} N_i}$, where R becomes the *round transmission rate*. If M goes to infinity, we obtain the LATR as $\frac{R}{E[N]}$ (bps/Hz).

We analyze and compare the following three two-hop relay schemes: 1) a two-hop relay scheme which does not adopt HARQ; 2) a two-hop relay scheme which uses HARQ without a delay constraint; and 3) a two-hop relay scheme which adopts HARQ with a delay constraint. All the two-hop relay schemes consist of two phases for transmission of a packet. In the first phase, the source transmits a packet to a relay. If the relay successfully decodes the packet, then the second phase starts. In the second phase, the relay transmits the packet to the destination.

III. System Analysis

A. Two-hop relay without HARQ

For a sufficiently long packet length, the mutual information of the link from node A to node B becomes $I_{\mathrm{AB}} = \log_2\left(1 + \left|h^l_{\mathrm{AB}}\right|^2 \frac{P_{\mathrm{A}}}{N_0}\right)$ for input with normal distribution and the outage probability for a given R is given by $P^{\mathrm{out}}_{\mathrm{AB}}(R) = \Pr\left[I_{\mathrm{AB}} < R\right] = \Pr\left[\left|h^l_{\mathrm{AB}}\right|^2 \frac{P_{\mathrm{A}}}{N_0} < \left(2^R - 1\right)\right]$. Since $X = \left|h^l_{\mathrm{AB}}\right|^2$ follows an exponential distribution, $P^{\mathrm{out}}_{\mathrm{AB}}(R)$ is expressed as the cumulative distribution function (CDF) of X, $1 - \exp(-\frac{2^R-1}{\rho_{\mathrm{AB}}})$, where ρ_{AB} is equal to $\frac{P_{\mathrm{A}}\sigma^2_{\mathrm{AB}}}{N_0}$. With HARQ, the packet is transmitted only once in the SR-link and the transmission in the RD-link is only possible when the transmission in the SR-link is successful. Therefore, the outage probability from the source to the destination is given by

$$P^{\mathrm{out}}_{\mathrm{SD}}(R_1,R_2) = \Pr[I_{\mathrm{SR}} < R_1] + (1-\Pr[I_{\mathrm{SR}} < R_1])\Pr[I_{\mathrm{RD}} < R_2] \tag{1}$$

$$= 1 - \exp\left(-\frac{2^{R_1}-1}{\rho_{\mathrm{SR}}}\right)\exp\left(-\frac{2^{R_2}-1}{\rho_{\mathrm{RD}}}\right). \tag{2}$$

For a successful packet transmission, at least two transmission rounds are required. In addition, the LATR of the two-hop relay without HARQ is given by

$$\mathcal{C}(R_1,R_2) = \frac{b}{T_1 + T_2(1-P^{\mathrm{out}}_{\mathrm{SR}}(R_1))} \tag{3a}$$

$$= \frac{1}{1/R_1 + \exp\left(-\frac{2^{R_1}-1}{\rho_{\mathrm{SR}}}\right)/R_2}, \tag{3b}$$

An optimization problem to maximize the LATR while guaranteeing a given outage probability constraint, ϵ, is given by.

$$\max_{R_1,R_2} \quad \mathcal{C}(R_1,R_2) = \frac{1}{1/R_1 + \exp\left(-\frac{2^{R_1}-1}{\rho_{\mathrm{SR}}}\right)/R_2}, \tag{4a}$$

$$\text{s.t.} \quad P^{\mathrm{out}}_{\mathrm{SD}}(R_1,R_2) = 1 - \exp\left(-\frac{2^{R_1}-1}{\rho_{\mathrm{SR}}}\right)\exp\left(-\frac{2^{R_2}-1}{\rho_{\mathrm{RD}}}\right) \le \epsilon, \tag{4b}$$

$$R_1, R_2 \ge 0. \tag{4c}$$

Both $P^{\mathrm{out}}_{\mathrm{SD}}(R_1,R_2)$ and $\mathcal{C}(R_1,R_2)$ are an increasing function of R_1 and R_2, i.e. $P^{\mathrm{out}}_{\mathrm{SD}}(R_1,R_2) < P^{\mathrm{out}}_{\mathrm{SD}}(R_1+\Delta_1,R_2+\Delta_1)$ and $\mathcal{C}(R_1,R_2) < \mathcal{C}(R_1+\Delta_1,R_2+\Delta_2)$ for any $\Delta_1,\Delta_2 > 0$. Optimal solutions are always on the boundary of the constraint (4b) and we prove this as follows:

Proof: Let $A = \{R_1,R_2 \ge 0 : P^{\mathrm{out}}_{\mathrm{SD}}(R_1,R_2) < \epsilon\}$. For $\forall R_1,R_2 \in A$, there exist $\Delta_1 > 0$ and $\Delta_2 > 0$ which satisfy $P^{\mathrm{out}}_{\mathrm{SD}}(R_1+\Delta_1,R_2+\Delta_2) = \epsilon$. Since $\mathcal{C}(R_1,R_2)$ is an increasing function, $\mathcal{C}(R_1, R_2) < \mathcal{C}(R_1+\Delta_1, R_2+\Delta_2)$. Therefore, all R_1 and R_2 which belong to A can not be optimal solutions $\square$.

By changing inequality (4b) into an equation: $1 - \exp\left(-\frac{2^{R_1}-1}{\rho_{\text{SR}}}\right)\exp\left(-\frac{2^{R_2}-1}{\rho_{\text{RD}}}\right) = \epsilon$, we rewrite this equation as a function $R_1(R_2)$: $R_1 = \log_2[1 + \frac{\rho_{\text{SR}}}{\rho_{\text{RD}}} - \rho_{\text{SR}}\ln(1-\epsilon) - \frac{\rho_{\text{SR}}}{\rho_{\text{RD}}}2^{R_2}]$. The optimization problem can be modified as follows:

$$\min_{R_2} \quad \mathcal{D}(R_2) = \frac{1}{\log_2\left(1 + \frac{\rho_{\text{SR}}}{\rho_{\text{RD}}} - \rho_1\ln(1-\epsilon) - \frac{\rho_1}{\rho_{\text{RD}}}2^{R_2}\right)} + \exp\left(\frac{2^{R_2}-1}{\rho_{\text{RD}}} + \ln(1-\epsilon)\right)/R_2, \tag{5a}$$

$$\text{s.t.} \quad 0 \le R_2 \le \log_2\left(1 - \rho_{\text{RD}}\ln(1-\epsilon)\right), \tag{5b}$$

where $\mathcal{D}(R_2)$ is a convex function in the range of $0 \le R_2 \le \log_2\left(1 - \rho_{\text{RD}}\ln(1-\epsilon)\right)$. See Appendix A. Therefore, we can find the optimal value of R_2, R_2^*, by using a golden section method [18].

B. Two-hop relay with HARQ

We first discuss the outage performance of a single link between node A and node B. If a Chase combining scheme is applied for a sufficiently long packet, the mutual information from node A and B after the kth HARQ round becomes expressed as

$$I_{\text{AB},k} = \log_2\left(1 + \sum_{l=1}^{k}\left|h_{\text{AB}}^{l}\right|^2\frac{P_{\text{A}}}{N_0}\right), \tag{6}$$

where N_0 denotes the one-sided noise spectral density. The outage probability after the kth HARQ round is given by

$$P_{\text{AB},k}^{\text{out}}(R) = \Pr\left[I_{\text{AB},k} < R\right] = \Pr\left[\sum_{l=1}^{k}\left|h_{\text{AB}}^{l}\right|^2\frac{P_{\text{A}}}{N_0} < \left(2^R - 1\right)\right].$$

Let $Z_l = \left|h_{\text{AB}}^{l}\right|^2\frac{P_{\text{A}}}{N_0}$. Z_l follows an exponential distribution with rate $\frac{N_0}{\sigma_{\text{AB}}^2 P_{\text{A}}}$ because $h_{\text{AB}} \sim \mathcal{C}(0, \sigma_{\text{AB}}^2)$. We let $X = \sum_{l=1}^{k} Z_l$ which is an Erlang-distributed random variable (RV) with shape k and rate $\lambda = \frac{N_0}{\sigma_{\text{AB}}^2 P_{\text{A}}}$, whose CDF is given by

$$F_X(x,k,\lambda) = 1 - \sum_{n=0}^{k-1} e^{-\lambda x}(\lambda x)^n/n! \tag{7}$$

$$= \sum_{n=0}^{\infty} e^{-\lambda x}(\lambda x)^n/n! - \sum_{n=0}^{k-1} e^{-\lambda x}(\lambda x)^n/n!$$

$$= \sum_{n=k}^{\infty} e^{-\lambda x}(\lambda x)^n/n! \tag{8}$$

Therefore, $P_{\text{AB},k}^{\text{out}}(R) = F_X(2^R - 1, k, \frac{N_0}{\sigma_{\text{AB}}^2 P_{\text{A}}})$. In addition, the probability that a packet is successfully decoded after the kth HARQ round is written as

$$P_{\text{AB},k}^{\text{suc}}(R) = P_{\text{AB},k-1}^{\text{out}}(R) - P_{\text{AB},k}^{\text{out}}(R). \tag{9}$$

In HARQ schemes, the maximum number of HARQ rounds can be denoted by L. The expected number of HARQ rounds per packet for given R and L is expressed as

$$E[N|R] = \sum_{k=1}^{L} k \cdot P_{\text{AB},k}^{\text{suc.}}(R) + L \cdot P_{\text{AB},L}^{\text{out}}(R) = \sum_{k=0}^{L-1} P_{\text{AB,k}}^{\text{out}}(R), \tag{10}$$

We now consider a two-hop relay case. The outage probability of the two-hop relay with HARQ after the L-th HARQ round is given by

$$P_{\text{SD},L}^{\text{out}}(R_1,R_2) = \sum_{r=1}^{L-1} P_{\text{SR},r}^{\text{suc}}(R_1)P_{\text{RD},(L-r)}^{\text{out}}(R_2) + P_{\text{SR},(L-1)}^{\text{out}}(R_1). \tag{11}$$

The average transmission rate over M packets becomes $\frac{Mb}{T_1\sum_{i=1}^{M}N_i^{(\text{SR})} + T_2\sum_{i=1}^{M}N_i^{(\text{RD})}}$, where $N_i^{(\text{SR})}$ and $N_i^{(\text{RD})}$ denote the number of HARQ rounds used for the i-th packet in the SR-link and the RD-link, respectively. Then, the LATR becomes $1/(E[N^{\text{SR}}|R_1]/R_1 + E[N^{\text{RD}}|R_1,R_2]/R_2)$ where $E[N^{\text{SR}}|R_1]$ and $E[N^{\text{RD}}|R_1,R_2]$ are expressed as

$$E[N^{\text{SR}}|R_1] = \sum_{l=1}^{L-1} l \cdot P_{\text{SR},l}^{\text{suc}}(R_1) + (L-1)P_{\text{SR},L-1}^{\text{out}}(R_1) = \sum_{l=0}^{L-2} P_{\text{SR},l}^{\text{out}}(R_1), \tag{12a}$$

$$E[N^{\text{RD}}|R_1,R_2] = \sum_{l_1=1}^{L-1}\left(P_{\text{SR},l_1}^{\text{suc}}(R_1)\sum_{l_2=0}^{L-l_1-1} P_{\text{RD},l_2}^{\text{out}}(R_2)\right), \tag{12b}$$

respectively. The term, $\sum_{l_2=0}^{L-l_1-1} P_{\text{RD},l_2}^{\text{out}}(R_2)$ in (12b) represents the average number of HARQ rounds in the RD-link when the SR-link uses l_1 HARQ rounds. The optimization problem finding the optimal R_1 and R_2 to maximize the LATR is formulated as **[Opt-HARQ]**

$$\max_{R_1,R_2} \quad \mathcal{C}(R_1,R_2) = \frac{1}{E[N^{\text{SR}}|R_1]/R_1 + E[N^{\text{RD}}|R_1,R_2]/R_2} \tag{13a}$$

$$\text{s.t.} \quad P_{\text{SD},L}^{\text{out}}(R_1,R_2) \le \epsilon, \quad R_1, R_2 \ge 0. \tag{13b}$$

We find the optimal R_1 and R_2 in infinite and finite L cases. In the infinite L case, R_1^* and R_2^* can be found analytically. But in the finite L case, we need a numerical search algorithm to find R_1^* and R_2^*.

1) Infinite L case: If L is infinite, (12a) can be derived as follows (See Appendix B):

$$E[N^{\text{SR}}|R_1] = \sum_{l=0}^{\infty} P_{\text{SR,l}}^{\text{out}}(R_1), \tag{14a}$$

$$= \exp\left(-\frac{2^{R_1}-1}{\rho_{\text{SR}}}\right)\sum_{l=0}^{\infty}\sum_{n=l}^{\infty}\left(\frac{2^{R_1}-1}{\rho_{\text{SR}}}\right)^n/n!$$

$$= \frac{2^{R_1}-1}{\rho_{\text{SR}}} + 1. \tag{14b}$$

In addition, we find the limit of (12b) for infinite L. The term in the summation is bounded by

$$P^{\text{suc}}_{\text{SR},l_1}(R_1)\sum_{l_2=0}^{L-l_1-1}P^{\text{out}}_{\text{RD},l_2}(R_2)\leq P^{\text{suc}}_{\text{SR},l_1}(R_1)\sum_{l_2=0}^{L}P^{\text{out}}_{\text{RD},l_2}(R_2). \quad (15)$$

For infinite L and finite l_1, it is clear that equality holds in (15). For infinite L and l_1, the right-hand side of (15) converges to zero since $\lim_{l_1\to\infty} P^{\text{suc}}_{\text{SR},l_1}(R_1)\to 0$ and $\sum_{l_2=0}^{L} P^{\text{out}}_{\text{RD},l_2}(R_2)$ is finite for finite R_1 and R_2. Therefore, we can say that equality holds in (15) for any l_1 value. For infinite L, (12b) can be derived as follows:

$$\begin{aligned}
E[N^{\text{RD}}|R_1,R_2] &= \sum_{l_1=1}^{\infty}\left(P^{\text{suc}}_{\text{SR},l_1}(R_1)\sum_{l_2=0}^{\infty}P^{\text{out}}_{\text{RD},l_2}(R_2)\right) && \text{(16a)}\\
&= \sum_{l_2=0}^{\infty}P^{\text{out}}_{\text{RD},l_2}(R_2)\sum_{l_1=1}^{\infty}P^{\text{suc}}_{\text{SR},l_1}(R_1)\\
&= \exp(-\frac{2^{R_2}-1}{\rho_{\text{RD}}})\sum_{l=0}^{\infty}\sum_{n=l}^{\infty}(\frac{2^{R_2}-1}{\rho_{\text{RD}}})^n/n!\\
&= \frac{2^{R_2}-1}{\rho_{\text{RD}}}+1. && \text{(16b)}
\end{aligned}$$

In (16a), we used $(L-l_1-1)\to\infty$ as $L\to\infty$. In (16b), we used $\sum_{l_1=1}^{\infty} P^{\text{suc}}_{\text{SR},l_1}(R_1)=1$. In (16b), we used the result of (14). Interestingly, $E[N^{\text{RD}}|R_1,R_2]$ is not dependent on R_2 in the infinite L case. Moreover, $P^{\text{out}}_{\text{SD},}(R)$ goes to zero as L goes to infinity. Therefore, the **Opt-HARQ** is formulated in the infinite L case as

$$\begin{aligned}
\max_{R_1,R_2} \quad & \mathcal{C}(R_1,R_2)=\frac{1}{\left(\frac{2^{R_1}-1}{\rho_{\text{SR}}}+1\right)/R_1+\left(\frac{2^{R_2}-1}{\rho_{\text{RD}}}+1\right)/R_2}\\
\text{s.t.} \quad & R_1,R_2\geq 0.
\end{aligned}$$

The above optimization problem can be reformulated as

$$\begin{aligned}
\min_{R_1,R_2} \quad & \mathcal{D}(R_1,R2)=\left(\frac{2^{R_1}-1}{\rho}+1\right)/R_1+\left(\frac{2^{R_2}-1}{\rho}+1\right)/R_2,\\
\text{s.t.} \quad & R_1,R_2\geq 0.
\end{aligned}$$

We find R_1^* and R_2^* which satisfy $\frac{\delta\mathcal{D}(R_1,R2)}{\delta R_1}|_{R_1=R_1^*}=0$ and $\frac{\delta\mathcal{D}(R_1,R2)}{\delta R_2}|_{R_2=R_2^*}=0$. These two equations are simplified to $\ln 2\cdot 2^{R_1^*}\cdot R_1^*-2^{R_1^*}+1-\rho_{\text{SR}}=0$ and $\ln 2\cdot 2^{R_2^*}\cdot R_2^*-2^{R_2^*}+1-\rho_{\text{RD}}=0$, respectively, that is, $\mathcal{D}(R_1,R2)$ has a unique minimum point since each equation has unique solution. Solutions of the two simplified equations are given by

$$\begin{aligned}
R_1^* &= \frac{W\left(\frac{\rho_{\text{SR}}-1}{e}\right)+1}{\ln 2},\\
R_2^* &= \frac{W\left(\frac{\rho_{\text{RD}}-1}{e}\right)+1}{\ln 2}, && \text{(17)}
\end{aligned}$$

respectively, where $W(x)$ is the Lambert W function. If we let $z=W(x)$, z is a unique solution of $x=z\exp(z)$. Interestingly, R_1^* (R_2^*) does not depend on the channel statistics in RD-link (SR-link). In order to compare the optimal rate selection method with a non-optimal method, we consider the following problem named as [**Eq-Rate**]

$$\begin{aligned}
\max_{R_1,R_2} \quad & \mathcal{C}(R_1,R2)=\frac{1}{\left(\frac{2^{R_1}-1}{\rho_{\text{SR}}}+1\right)/R_1+\left(\frac{2^{R_2}-1}{\rho_{\text{RD}}}+1\right)/R_2},\\
\text{s.t.} \quad & R_1=R_2\geq 0,
\end{aligned}$$

where the *round transmission rates* of two links are identical to each other. We obtain the following equation: $\ln 2\cdot 2^{R^*}\cdot R^*-2^{R^*}+(1-\frac{2}{1/\rho_{\text{SR}}+1/\rho_{\text{RD}}})=0$. Therefore, $R_1^*=R_2^*=R^*=\frac{W\left(\left(\frac{2}{1/\rho_{\text{SR}}+1/\rho_{\text{RD}}}-1\right)/e\right)+1}{\ln 2}$.

2) Finite L case: If L is finite, the optimization problem, **Opt-HARQ**, becomes too complicated to be tracked. At first, $\mathcal{C}(R_1,R2)$ is neither an increasing nor a decreasing function and may have multiple local maximum points or minimum points. Moreover, $E[N^{\text{RD}}|R_1,R_2]$ depends on R_1 and R_2. $P^{\text{out}}_{\text{SD},L}(R)$ has a non-zero value in the finite L case. However, we know $P^{\text{out}}_{\text{SD},L}(R_1,R_2)$ is an increasing function since the larger R_i requires the larger mutual information to be successfully decoded. Empirically, we assume that $\mathcal{C}(R_1,R2)$ becomes a concave function within $P^{\text{out}}_{\text{SD},L}(R_1,R_2)\leq\epsilon$ for a small ϵ value such as $\epsilon\leq 0.02$. By using this assumption, we propose a heuristic search algorithm to find R_1^* and R_2^*. At first, we fix a ratio $\alpha=R_1/R_2$. For a fixed α, we find $R_1^{\epsilon}(\alpha)$ such that $P^{\text{out}}_{\text{SD},L}(R_1^{\epsilon},\alpha R_1^{\epsilon})=\epsilon$. Since $P^{\text{out}}_{\text{SD},L}$ is an increasing function of R_1, $R_1^{\epsilon}(\alpha)$ is easily found. We can find $R_1^{\max}(\alpha)=\arg_{R_1}\max\mathcal{C}(R_1,\alpha R_1)$ by using a golden section method in the range, $0\leq R_1\leq R_1^{\epsilon}(\alpha)$ and then we also can find the optimal R_1 for a given α such that $R_1^*(\alpha)=\min(R_1^{\max}(\alpha),R_1^{\epsilon}(\alpha))$ since $R_1^{\max}(\alpha)>R_1^{\epsilon}(\alpha))$ does not satisfy the outage probability constraint. We repeat this process for different α values by changing α gradually. Finally, we search α^* which maximizes $\mathcal{C}(R_1^*(\alpha),R_1^*(\alpha))$ among many α values.

IV. Performance Comparisons

We evaluate the performance of a two-hop relay network for varying the placement of the relay node. The average received SNR is modeled as a function of distance d [19]: $\rho_{\text{AB}}(d)=\frac{P_{\text{A}}}{N_0}\sigma^2_{\text{AB}}=\frac{P_{\text{A}}}{N_0}K\left(\frac{d_0}{d}\right)^{\gamma}$, where the channel variance σ^2_{AB} is determined by the path loss, K is a unitless constant depending on the antenna characteristics and the average channel attenuation, d_0 denotes the reference distance, and γ is the path-loss exponent. As a representative simulation example, we choose d_0 =10 m, $\gamma=3$, $K=1$, and $P_{\text{S}}/N_0=60$ dB. Let d_{AB} denote the distance between nodes A and B. Although d_{SD} may vary in a very wide range, we choose $d_{\text{SD}}=800$ m corresponding to a small average S-D link SNR of 2.907 dB, which requires a relay node for achieving a good LATR.

Fig. 1 shows the LATR versus d_{SR} for the two-hop relay without HARQ, the two-hop relay with HARQ with infinite L, and the two-hop relay with HARQ with $L=3,4$ and 5 and $\epsilon=10^{-2}$. Note that the two-hop relay without HARQ is identical to the the two-hop relay with HARQ with $L=2$. The LATR is close to that in the infinite L case as L goes to infinite. The non-HARQ two-hop relay scheme yields very poor performance. The gap between the optimal rate selection

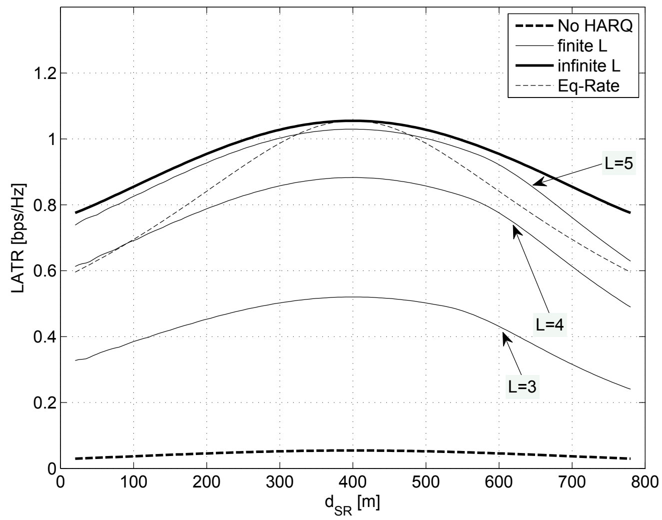

Fig. 1. LATR versus $d_{\rm SR}$ for the non-HARQ, HARQ with an infinite L and HARQ with $L = 3, 4$ and 5 when $d_{\rm SD} = 800$ m and $\epsilon = 10^{-2}$.

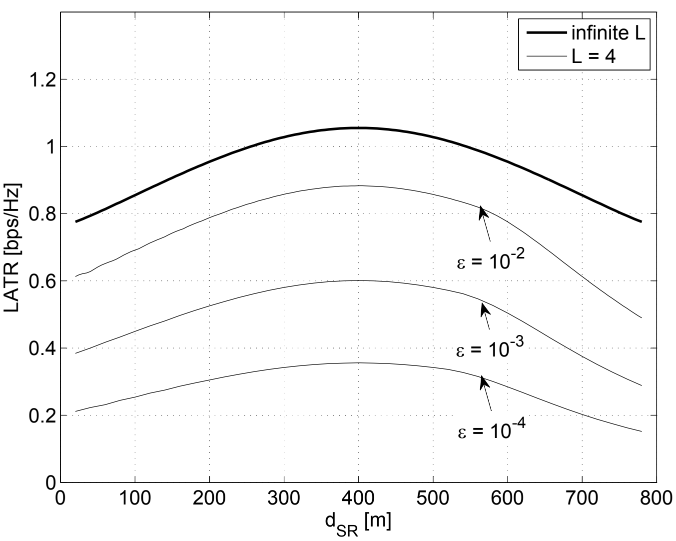

Fig. 2. LATR versus $d_{\rm SR}$ for the HARQ with an infinite L and HARQ with a finite $L = 4$ when $\epsilon = 10^{-2}, 10^{-3}$ and 10^{-4} and $d_{\rm SD} = 800$ m.

method and the **Eq-Rate** scheme ($R_1 = R_2$) in the infinite L case increases as the relay node becomes close to the source node or the destination node.

Fig. 2 shows the LATR versus $d_{\rm SR}$, for the HARQ with an infinite L and HARQ with a finite number of $L = 4$ for three different ϵ values of $10^{-2}, 10^{-3}$ and 10^{-4}. For HARQ with a finite L, the outage probability constraint gives a critical effect on the LATR performance. With a fixed L, the LATR peformance degrades as ϵ decreases.

Fig. 3 shows the LATR versus L for the HARQ with finite L with $\epsilon = 10^{-2}, 10^{-3}$ and 10^{-4} when $\rho_{\rm SR} = \rho_{\rm RD} = 10$ [dB]. We can observe that for a fixed ϵ the LATR of HARQ with a finite L is saturated to that of the HARQ with an infinite L as L increases. Moreover, as ϵ decreases the larger L is required to yield the same LATR performance. However, note

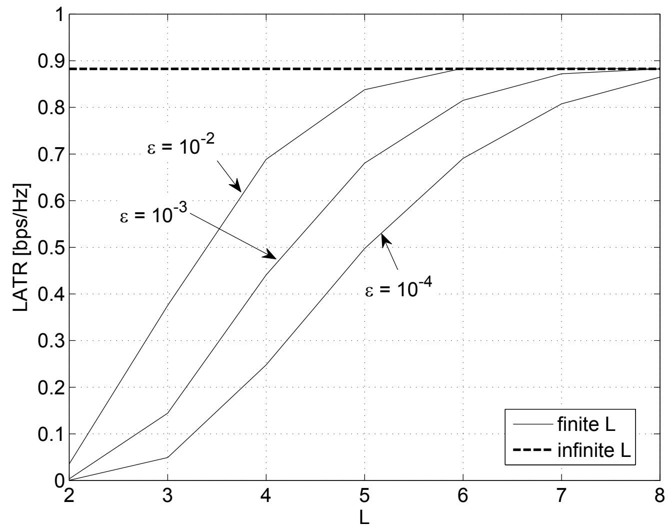

Fig. 3. LATR versus L for the HARQ with a finite L with $\epsilon = 10^{-2}, 10^{-3}$ and 10^{-4} when $\rho_{\rm SR} = \rho_{\rm RD} = 10$ [dB].

that the LATR of the infinite L case is not an upper bound of LATRs of finite L cases. The infinite L case does not allow any decoding failure, therefore, it is possible for a few packets to require a large number of HARQ rounds to be successfully decoded. If L is finite, although some packets are not decoded successful, the number of HARQ rounds is limited to L. This makes the finite L cases yield better LATR than that of the infinite L case. However, for a small ϵ, this amount of LATR gain is very negligible. Therefore, we do not focus on the control of L.

V. Conclusions

We proposed to find the *round transmission rates* of the SR-link and the RD-link of a two-hop relay network with CC-type HARQ in order to maximize the LATR. For the two-hop relay without HARQ, the optimal *round transmission rates* can be found by using the convexity of the LATR at the constraint boundary. For the two-hop relay with HARQ in the infinite L and the finite L cases, we also found the optimal *round transmission rates*. In the infinite L case, the optimal *round transmission rates* are expressed as Lambert W functions and each rate only depends on the channel statistics of its own hop. In the finite L case, we proposed a numerically searching algorithm to find the *round transmission rates*. The LART performance is saturated to that of the infinite L case as L goes to infinity for a small ϵ value. Our proposed rate selection method outperforms the **Eq-Rate** method in the infinite L case and the gain increases as the relay node becomes close to either the source node or the destination node.

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APPENDIX A
PROOF OF THE CONVEXITY FOR $\mathcal{D}(R_2)$

Let $A(R_2) = \log_2\left(1+\frac{\rho_{\text{SR}}}{\rho_{\text{RD}}} - \rho_{\text{SR}}\ln(1-\epsilon) - \frac{\rho_{\text{SR}}}{\rho_{\text{RD}}}2^{R_2}\right)$ and $B(R_2) = \exp\left(\frac{2^{R_2}-1}{\rho_{\text{RD}}} + \ln(1-\epsilon)\right)/R_2$. Then, $\mathcal{D}(R_2) = 1/A(R_2) + B(R_2)$. We note that $A(R_2)$ is a decreasing function of R_2 and $A(R_2) \geq 0$ within the constraint $0 \leq R_2 \leq \log_2(1-\rho_{\text{RD}}\ln(1-\epsilon))$. The second derivative of $\mathcal{D}(R_2)$ is expressed as

$$\mathcal{D}''(R_2) = \frac{-A''(R_2)\cdot A(R_2) + 2(A'(R_2))^2}{(A(R_2))^3} + B''(R_2), \tag{A.1}$$

where

$$A'(R_2) = \frac{-\rho_{\text{SR}}/\rho_{\text{RD}}\cdot\ln(2)\cdot 2^{R_2}}{1+\rho_{\text{SR}}/\rho_{\text{RD}} - \rho_{\text{SR}}\ln(1-\epsilon) - \rho_{\text{SR}}/\rho_{\text{RD}}\cdot 2^{R_2}},$$

$$A''(R_2) = \frac{-\left(\rho_{\text{SR}}/\rho_{\text{RD}}\cdot\ln(2)\cdot 2^{R_2}\right)^2}{\left(1+\rho_{\text{SR}}/\rho_{\text{RD}} - \rho_{\text{SR}}\ln(1-\epsilon) - \rho_{\text{SR}}/\rho_{\text{RD}}\cdot 2^{R_2}\right)^2},$$

$$B''(R_2) = B(R_2)\times \left(\left(\frac{\ln(2)\cdot 2^{R_1}}{\rho_{\text{RD}}^2} - \frac{1}{R_2}\right)^2 + \left(\frac{(\ln(2))^2\cdot 2^{R_1}}{\rho_{\text{RD}}^2} + \frac{1}{R_2^2}\right)\right).$$

By substituting $A(R_2), A'(R_2), A''(R_2)$, and $B''(R_2)$, we can know that $\mathcal{D}''(R_2)$ is always greater than 0 within the constraint $0 \leq R_2 \leq \log_2(1-\rho_{\text{RD}}\ln(1-\epsilon))$.

APPENDIX B
DERIVATION OF (14)

$$\begin{aligned}
E[N^{\text{SR}}|R_1] &= \sum_{l=0}^{\infty} P_{\text{SR,l}}^{\text{out}}(R_1), \\
&= \exp\left(-\frac{2^{R_1}-1}{\rho_{\text{SR}}}\right)\sum_{l=0}^{\infty}\sum_{n=l}^{\infty}\left(\frac{2^{R_1}-1}{\rho_{\text{SR}}}\right)^n/n! \\
&= \exp\left(-\frac{2^{R_1}-1}{\rho_{\text{SR}}}\right)\sum_{n=0}^{\infty}\sum_{l=0}^{n}\left(\frac{2^{R_1}-1}{\rho_{\text{SR}}}\right)^n/n! \\
&= \exp\left(-\frac{2^{R_1}-1}{\rho_{\text{SR}}}\right)\sum_{n=0}^{\infty}(n+1)\left(\frac{2^{R_1}-1}{\rho_{\text{SR}}}\right)^n/n! \\
&= \exp\left(-\frac{2^{R_1}-1}{\rho_{\text{SR}}}\right)\sum_{n=0}^{\infty}n\left(\frac{2^{R_1}-1}{\rho_{\text{SR}}}\right)^n/n! \\
&\quad + \exp\left(-\frac{2^{R_1}-1}{\rho_{\text{SR}}}\right)\sum_{n=0}^{\infty}\left(\frac{2^{R_1}-1}{\rho_{\text{SR}}}\right)^n/n! \\
&= \exp\left(-\frac{2^{R_1}-1}{\rho_{\text{SR}}}\right)\sum_{n=1}^{\infty}\left(\frac{2^{R_1}-1}{\rho_{\text{SR}}}\right)^n/(n-1)! + 1 \\
&= \exp\left(-\frac{2^{R_1}-1}{\rho_{\text{SR}}}\right)\left(\frac{2^{R_1}-1}{\rho_{\text{SR}}}\right)\sum_{n=0}^{\infty}\left(\frac{2^{R_1}-1}{\rho_{\text{SR}}}\right)^n/n! + 1 \\
&= \frac{2^{R_1}-1}{\rho_{\text{SR}}} + 1.
\end{aligned}$$

REFERENCES

[1] D. Chase, "Code combining - a maximum-likelihood decoding approach for combining an arbitrary number of noisy packets," in *IEEE Trans. Commun.,* vol. COM-33, no. 5, pp. 385-393, May 1985.

[2] S. Lin and P. S. Yu, "A hybrid ARQ scheme with parity retransmission for error control of satellite channels," in *IEEE Trans. Commun.,* vol. COM-30, no. 7, pp. 1701-1719, July 1982.

[3] Y.-M. Wang and S. Lin, "A modified selective-repeat type-II hybrid ARQ system and its performance analysis," in *IEEE Trans. Commun.,* vol. COM-31, no. 5, pp. 593-608, May 1983.

[4] G. Caire and D. Tuninetti, "The throughput of hybrid-ARQ protocols for the Gaussian collision channel," in *IEEE Trans. Information Theory,* vol. 47, no. 5, pp. 1971-1988, July 2001.

[5] P. Wu and N. Jindal, "Performance of hybrid-ARQ in block-fading channels : A fixed outage probability analysis," in *IEEE Trans. Commun.,* vol. 58, no. 4, pp. 1129-1141, Apr. 2010.

[6] H. Zheng and H. Viswanathan, "Optimizaing the ARQ performance in downlink packet data systems with scheduling," in *IEEE Trans. Wireless Commun.*, vol. 4, no. 2, pp. 495-506, Mar. 2005.

[7] D. Kim, B. C. Jung, D. K. Sung and H. Yoon, "Optimal modulation and coding scheme selection in cellular networks with hybrid-ARQ error control," in *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 5195-5201, Dec. 2008.

[8] R. Aguero, J. Choque, and L. Munoz, "On the relay-based coverage extension for non-conventional multi-hop wireless networks," in *Proc. IEEE Int. Conf. in Commun. (ICC),* Glasgow, Scotland, June 2007.

[9] B. Zhao and M. C. Valenti, "Practical relay networks: A generalization of hybrid-ARQ," in *IEEE J. Sel. Areas in Commun.*, vol. 23, no. 1, pp. 7-18, Jan. 2005.

[10] T. Tabet, S. Dusad, and R. Knopp, "Diversity-multiplexing-delay trade-off in half-duplex ARQ relay channels," in *IEEE Trans. Inform. Theory,* vol. 53, no. 10, pp. 3797-3805, Oct. 2007.

[11] I. Stanojev, O. Simeone, Y. Bar-Ness, and D. H. Kim, "Energy Efficiency of Non-Collaborative and Collaborative Hybrid-ARQ Protocl," in *IEEE Trans. Wireless Commun.,* vol. 8, no. 1, pp. 326-335, Jan. 2009

[12] S. Lee, W. Su, S. Batalama, and J. D. Matyjas, "Cooperative Decode-and-Forward ARQ Relaying Performance Analysis and Power Optimization," in *IEEE Trans. Wireless Commun.,* vol. 9, no. 8, pp. 2632-2642, Jul. 2010.

[13] S. H. Kim, S. J. Lee, and D. K. Sung, "Optimal Rate Selelction Scheme in a Two-hop Relay Network Adopting Chase Combining HARQ in Rayleigh Block-fading Channels," in *IEEE Trans. Vehicular Tech.,* vol. 60, no. 9, pp. 1640-1645, Jul. 2010.

[14] IEEE 802.16j Draft Amendment, Part 16: Air interface for fixed and mobile broadband wireless access systems, multihop relay specification, IEEE 802.16j/D9, Feb. 2009.

[15] O. Oyman, N. J. Laneman, and S. Sandhu, "Multihop relaying for broadband wireless mesh networks: from theory to practice," in *IEEE Communications Magazine* , vol. 45, no. 11, pp. 116?122, 2007.

[16] S. H. Kim, S. M. Kim, and D. K. Sung "Combining Data Aggregation and HARQ Schemes to Guarantee Quality-of-Services in Uplink Relay-based Networks," in *Proc. IEEE International Conference on Communications,* Cape Town, South Africa, May. 2010.

[17] S. H. Kim, H. Jin, and D. K. Sung "Optimizing the Persistent Scheduling in Two-hop Relay Networks," in *Proc. IEEE Wireless Communications and Networking Conference* , Cancun, Mexico, Mar. 2011.

[18] J. Kiefer, "Sequential minimax search for a maximum," *Proc. the American Mathematical Society*, vol. 4. no. 3, pp. 502-506, June 1953.

[19] A. Goldsmith, *Wireless Communications*, Cambridge, 2005.