Throughput Analysis fot Two-Hop Relay Mobile Ad Hoc Networks with Receiver Probing

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***Abstract*—Available works either explore the order sense capacity scaling laws or derive closed-form throughput results for mobile ad hoc networks (MANETs) where a transmitter randomly probes only once a neighboring node for possible transmission. Obviously, such single probing strategy may result in a significant waste of the precious transmission opportunities in highly dynamic MANETs since the randomly selected node may already get the packets that the transmitter hopes to deliver. In this paper, we consider a two-hop relay MANET where each transmitter may conduct multiple rounds of probing so as to identify a possible receiver. We first develop closed-form expressions for per node throughput capacity in such probing-based network, with a careful consideration of the time cost taken to probe for an eligible receiver in each time slot. Extensive numerical results are further presented to explore the possible maximum per node throughput capacity, the corresponding optimum setting of probing round limit, and also their relationships with the network control parameters, like the probing time limit, the redundancy limit and the number of users, etc.**

I. Introduction

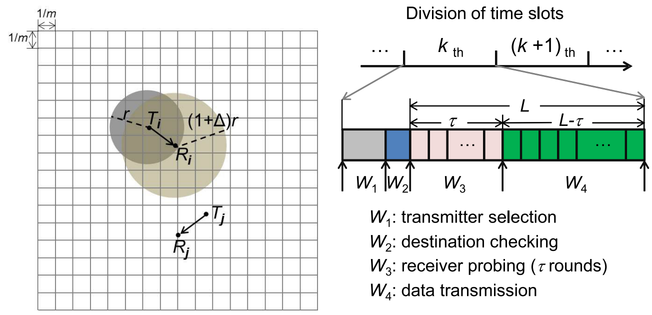
Mobile ad hoc networks (MANETs) in which mobile nodes communicate to each other via wireless links without any pre-existing infrastructure support or centralized management, hold great promise for lots of applications, such as the last mile wireless internet, vehicular ad hoc networks, disaster relief, etc. However, the lacking of network capacity theory characterizing the maximum achievable throughput between all node pairs, is still a challenging roadblock for the massive development and commercialization of such networks [1].

A lot of research efforts have been dedicated to exploring the order sense capacity scaling laws in MANETs. Grossglauser and Tse [2] showed that by adopting a two-hop relay algorithm, it is possible to achieve a $\Theta(1)$ per node throughput for MANETs with i.i.d. mobility. Later, it was proved that the constant throughput can also be achieved under other mobility models, like random walk model [3], Brownian motion model [4] and even the restricted mobility [5]. More recently, Neely *et al.* [6] showed that in a cell-partitioned network with the i.i.d. mobility, the $O(1/\sqrt{n})$ and $O(\frac{1}{n \log n})$ throughput are also achievable for each node by introducing redundant copies in the packet delivery process. Li *et al.* [7] also considered a cell-partitioned network which is first divided into $n^{2\alpha}$ cells and each cell is further divided into squares of area $n^{-2\beta}$. The authors showed that the throughput there is of order $O(n^{\beta-\alpha-\frac{1}{2}})$ and $\Omega(n^{2\beta-\alpha-1})$ if each node can only move within its own cell. Ciullo *et al.* in [8] proved that the throughput is $\Theta(mR^2/n)$ under the correlated mobility, where m is the number of clusters and R is the radius of a cluster region.

By now, some closed-form throughput capacity results have also been reported in literature. Neely *et al.* [6] derived the network capacity for a cell partitioned MANET with i.i.d. mobility, and showed that as the network size scales up the per node throughput capacity there remains constant for a given node density. The capacity results in [6] was later extended to a delay-tolerant MANET with general Markovian mobility [9]. In both [6] and [9], a transmitter in a cell randomly selects a neighboring node as the receiver. Liu *et al.* [10], [11] developed closed-form throughput results for two-hop relay based MANETs with limited packet redundancy and flexible power control.

Note that in all above works whenever a node getting an opportunity to transmit, it randomly probes only once a neighboring node for possible transmission, which may result in a significant waste of the precious transmission opportunities in highly dynamic MANETs since the randomly selected node may already get the packets that the transmitter hopes to deliver. In this paper, we consider a two-hop relay MANET where each transmitter may conduct multiple rounds of probing so as to identify a possible receiver. It is also noticed that some results have already been reported for the throughput capacity of such probing-based MANETs [12], [13]. However, the data transmission time in [12], [13] is assumed to be independent of the time taken to identify an eligible receiver, i.e., no matter how many rounds of probing are conducted by a transmitter, the remaining data transmission time remains unchanged.

In this paper, we consider a more practical scenario where the total time available for both receiver probing and data transmission is limited during a node meeting, and develop closed-form expressions for per node throughput capacity with a careful consideration of the time cost taken to probe for an eligible receiver. Based on the new throughput capacity result, extensive numerical results are further provided to explore the possible maximum per node throughput capacity, the corresponding optimum setting of probing round limit, and also their relationships with the network control parameters, like the probing time limit, the redundancy limit and the



(a) Cell partitioned network and the Protocol interference model.

(b) Division of a time slot.

Fig. 1. Illustration of system models.

number of users, etc.

The rest of this paper is organized as follows. Section II introduces the system models, time slot division and routing schemes considered in the paper. We derive closed-form expressions for per node throughput capacity in Section III, and present numerical results to explore the possible maximum throughput capacity and optimum probing round limit in Section IV. Finally, we conclude the whole paper in Section V.

II. System Models and Routing Scheme

A. System Models

Similar to [11], we assume that n nodes move independently in a square with unit area. As shown in Fig. 1a, the network area is evenly divided into $m \times m$ cells, each cell of side length $1/m$. Time is assumed to be divided into slots of equal duration, and the n nodes move among the m^2 cells according to the i.i.d. mobility model [6]. Initially, the n nodes are randomly distributed. At time slot 0, a node randomly and uniformly selects a cell from the m^2 cells with probability $1/m^2$ independent of other nodes, and moves to the selected cell at time slot 1. The node repeats the same process in every subsequent time slot. One can see that under the i.i.d. model, the network topology varies dramatically and can never be predicted. We adopt the permutation traffic [2], [8], where each node is not only the source of its locally generated traffic flow but also the destination of a flow originated from another node.

The Protocol model first introduced in [14] is adopted here to address the interference issue among simultaneous link transmissions. Suppose at time slot t, node T_i is transmitting to node R_i, as illustrated in Fig. 1a. If we denote by r the per node transmission range and denote by $d_t(T_i, R_i)$ the distance between T_i and R_i at time slot t, then we have $d_t(T_i, R_i) \le r$. In order to guarantee the data transmission from T_i to R_i to be successful, according to the Protocol model, for any other simultaneous transmitting node say T_j $(j \neq i)$, we should have $d_t(T_j, R_i) \ge (1 + \Delta) d_t(T_i, R_i)$. Here $\Delta > 0$ is a protocol defined guard factor so as to prevent other nodes from transmitting simultaneously at too close a distance.

We adopt the transmission-group based MAC (medium access control) scheme to schedule simultaneous link transmissions in the cell partitioned network [11], [15]. We assume that a whole time slot will be allocated only for data transmissions in one-hop range, and a node in a cell can only transmit to nodes in one of the eight adjacent cells or the same cell. Therefore, the per node transmission range r can be determined as $r = \sqrt{8}/m$. For the transmission-group based scheduling scheme, the distance parameter α, which defines the vertical and horizontal distances between active cells in a transmission-group, can be accordingly determined as $\alpha = \min\{\lceil (1+\Delta)\sqrt{8} + 2 \rceil, m\}$ [11].

B. Division of Time Slots

In order to enable an efficient utilization of limited channel bandwidth, the probing technique is adopted for identifying an eligible receiver in each time slot [13]. Under the receiver probing technique, as shown in Fig. 1b that each time slot is divided into four sub-slots: W_1, W_2, W_3 and W_4. Specifically, in sub-slot W_1, nodes in an active cell contend to become the transmitter according to a DCF-style scheme as introduced in [11], where each node starts its back-off counter with a randomly selected seed, and the node whose counter is the first to become zero announces itself as the transmitter. Sub-slot W_2 is used for destination checking, where the destination node of the flow originated from the selected transmitter will reply to the transmitter by broadcasting a message if it is in the one-hop neighborhood. If no reply is heard in sub-slot W_2, the transmitter will proceed to conduct at most τ rounds of receiver probing so as to identify an eligible receiver in sub-slot W_3; during each probing round the transmitter randomly selects a neighboring node to check whether it can deliver out some packets. Sub-slot W_4 is reserved for packets transmission from the transmitter to the receiver; if the transmitter fails to find an eligible receiver in W_3, it remains idle in W_4.

As shown in Fig. 1b, the total time allocated for receiver probing and data transmission in a time slot, i.e., sub-slots W_3 and W_4, has fixed duration of L mini-slots, where the first τ mini-slots are reserved for receiver probing. Since we consider a limited channel bandwidth in this paper, without loss of generality, we assume that a bundle of $L - \tau$ packets can be transmitted during sub-slot W_4. For the case that the destination node replies to notify the transmitter in sub-slot W_3, we further assume that the transmitter will also transmit a bundle of $L - \tau$ bits, so as to simplify the analysis.

C. Probing-Based Two-Hop Relay

We consider the probing-based two-hop relay algorithm 2HR-(τ, f) proposed in [12], [13], where each bundle (of $L-\tau$ packets) is delivered to at most f distinct relay nodes, and a transmitter conducts at most τ rounds of probing to select an eligible receiver when its destination node is not in the one-hop neighborhood. Since there are in total n distinct flows in the network, without loss of generality, we focus on a tagged flow hereafter and denote its source and destination by S and D, respectively.

2HR-(τ, f) Algorithm: every time S announces to become the transmitter in a time slot, it operates as follows:

Step 1: If S hears the reply from D in sub-slot W_2, S delivers a bundle of $L-\tau$ packets to D in sub-slot W_4 (source-to-destination transmission);

Step 2: Otherwise, S randomly performs one of the following operations with equal probability:

- source-to-relay transmission: S randomly selects a one-hop neighboring node V to see whether V carries a copy for the head-of-line bundle at S. If so, S proceeds to the next round of probing; otherwise, S stops probing and sends to V a new copy for its head-of-line bundle in sub-slot W_4.
- relay-to-destination transmission: for each randomly selected neighboring node V, S checks whether it carries a bundle that V is currently requesting for. If so, S stops probing and delivers the bundle to V in W_4; otherwise, S proceeds to the next round of probing.

For both Step 1 and Step 2 of the 2HR-(τ, f) algorithm, packets are transmitted and received in bundles where each bundle contains $L - \tau$ packets. It is noticed that in Step 2, S will conduct at most τ rounds of receiver probing, and at most f copies will be distributed for each bundle.

One can see from Step 2 of the 2HR-(τ, f) algorithm that by allowing more rounds of probing (i.e., with a bigger τ), the probability that a transmitter can find an eligible receiver in a time slot is also bigger; however, due to the time limitation for receiver probing and data transmission, a smaller number of packets can be transmitted from the transmitter to the receiver. Therefore, for a MANET adopting the 2HR-(τ, f) algorithm, the probing round limit τ should be carefully tuned so as to maximize the per node throughput capacity there.

III. Throughput Capacity

Before proceeding to derive closed-form expressions for the per node throughput capacity in a MANET adopting the 2HR-(τ, f) algorithm, we first introduce the data delivery process from the source to the destination. It is noticed that in the 2HR-(τ, f) based MANETs, data is transmitted and received in bundles. After a bundle is generated at the source, it typically experiences two service processes before its arrival at the destination, i.e., the dispatching process at the source and the receiving process at the destination.

We focus on the dispatching process first. Consider a time slot and a bundle say B at the source S of the tagged flow. Suppose there are already k copies (including the original one at S) of B in the network when the destination D starts to request for B, $1 \le k \le f + 1$. Then, one can see that in the next time slot, the probability that S will deliver out a new copy for B and the probability that S will directly send B to D, depend only on the current network state, i.e., the current spatial distribution of nodes in the network and the distribution of the $k - 1$ bundle copies among the $n - 2$ nodes. Since the dispatching process of each copy depends only on the network state in the current time slot (i.e., independent of the network states in previous time slots) and at most f copies will be distributed for the bundle B, the dispatching process of B at S can be modeled by a finite-state absorbing Markov chain.

Similarly, one can see that for the receiving process of the bundle B, the probability that D will receive B in the next time slot, also depends only on the network state in the current time slot. Therefore, the receiving process of B at D can also be defined by a finite-state absorbing Markov chain.

Given that there are k copies of B in the network when D starts to request for B, $1 \le k \le f+1$, if we denote by $X_S(k)$ the time it takes S to finish the copy dispatching for B and denote by $X_D(k)$ the time it takes D to receive B, then we have the following lemma.

Lemma 1: For any $1 \le k \le f$, the mean value of $X_S(k)$ and $X_D(k)$, i.e., $\mathbb{E}\{X_S(k)\}$ and $\mathbb{E}\{X_D(k)\}$, satisfy

$$\mathbb{E}\{X_S(k)\} < \mathbb{E}\{X_S(k+1)\} \tag{1}$$

$$\mathbb{E}\{X_D(k)\} > \mathbb{E}\{X_D(k+1)\} \tag{2}$$

Lemma 2: If we denote by $\overline{X}_S$ the average bundle dispatching time and by $\overline{X}_D$ the average bundle receiving time taken over all bundles of the tagged flow, then we have

$$\mathbb{E}\{X_S(1)\} \le \overline{X}_S \le \mathbb{E}\{X_S(f+1)\} \tag{3}$$

$$\mathbb{E}\{X_D(f+1)\} \le \overline{X}_D \le \mathbb{E}\{X_D(1)\} \tag{4}$$

Lemma 3: For any $1 \le f \le f_0$ and $1 \le \tau \le \tau_0$, we have

$$\mathbb{E}\{X_S(f+1)\} \le \mathbb{E}\{X_D(f+1)\} \tag{5}$$

where

$$f_0 = \left\lfloor \frac{(n-1)p_2}{p_2 + 2(n-2)p_1} \right\rfloor \tag{6}$$

$$\tau_0 = \min\left\{L, \left\lfloor \frac{n-1-f}{f^2} - \frac{2(n-2)p_1}{p_2 \cdot f} \right\rfloor\right\} \tag{7}$$

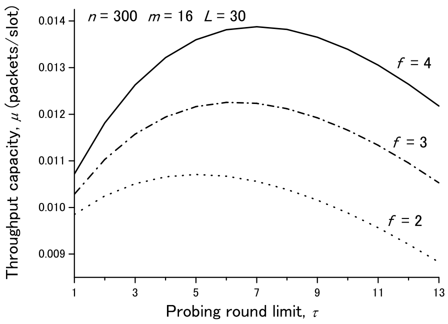
$$p_1 = \frac{1}{\alpha^2}\left\{\frac{9n-m^2}{n(n-1)} - \left(\frac{m^2-1}{m^2}\right)^{n-1}\frac{8n+1-m^2}{n(n-1)}\right\} \tag{8}$$

$$p_2 = \frac{1}{\alpha^2}\left\{\frac{m^2-9}{n-1}\left(1-\left(\frac{m^2-1}{m^2}\right)^{n-1}\right) - \left(\frac{m^2-9}{m^2}\right)^{n-1}\right\} \tag{9}$$

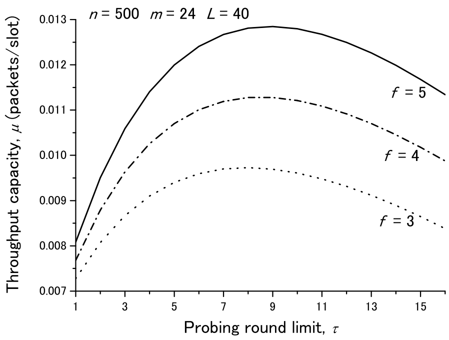
Based on (8) and (9), one can easily verify that for almost all (n, m) settings, we always have $p_1 < \frac{p_2}{2} \cdot \frac{n-3}{n-2} < \frac{p_2}{2}$. Therefore, the right-hand side in (6) satisfies $\frac{(n-1)p_2}{p_2+2(n-2)p_1} > 1$, and thus $f_0 \ge 1$.

Combining (5) with the results in Lemmas 1 and 2, one can see that for any $f \in [1, f_0]$ and $\tau \in [1, \tau_0]$, the smallest average bundle receiving time at D is always bigger than the biggest average bundle dispatching time at S. Therefore, we have the following theorem.

Theorem 1: Given that in each time slot the time allocated for receiver probing and data transmission is fixed as L mini-slots among which the first τ mini-slots are reserved for receiver probing, then the per node throughput capacity μ (packets/slot) in the 2HR-(τ, f) based MANETs can be

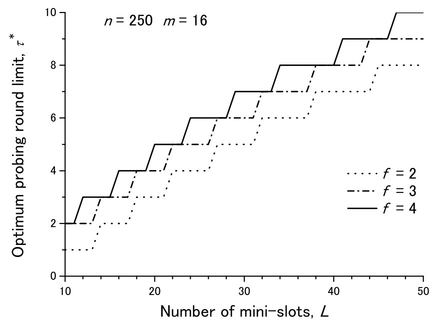


(a) Network scenario $(n = 300, m = 16, L = 30)$.

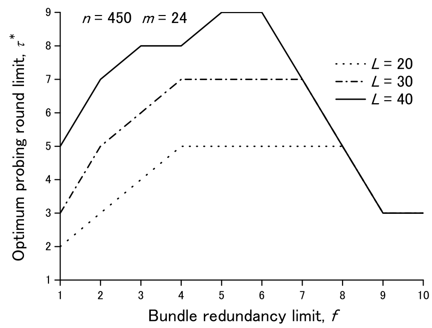


(b) Network scenario $(n = 500, m = 24, L = 40)$.

Fig. 2. Per node throughput capacity μ vs. probing round limit τ.



(a) τ^* vs. L.



(b) τ^* vs. f.

Fig. 3. Optimum probing round limit τ^* vs. the number of mini-slots L and the bundle redundancy limit f.

determined as

$$\mu = p_1(L-\tau) + \frac{f \cdot (m^2-9)^{n-2} \cdot (L-\tau)}{2\alpha^2(n^2-3n+2)m^{2n-2}} \sum_{k=0}^{n-3} \binom{n-1}{k+2} \cdot \frac{9^{k+2}-8^{k+2}}{(m^2-9)^k} \cdot \frac{(k+1)^\tau (n-2)^\tau - k^\tau (n-2-f)^\tau}{(n-2)^{\tau-1}(k+1)^{\tau-1}(n-2+kf)} \quad (10)$$

for any $f \in [1, f_0]$ and $\tau \in [1, \tau_0]$, where f_0, τ_0 and p_1 are given in (6), (7) and (8), respectively.

The proofs of Lemmas 1, 2, 3 and Theorem 1 are similar to that in [13] and are omitted here due to space limit.

One can observe from (10) that the probing round limit τ is actually very complicatedly involved in the calculation of per node throughput capacity μ.

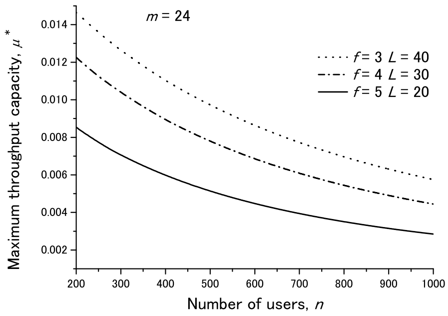
IV. Numerical Results

Based on the per node throughput capacity result in Section III, in this section we analytically explore the maximum per node throughput capacity and the corresponding optimum setting of probing round limit to achieve it.

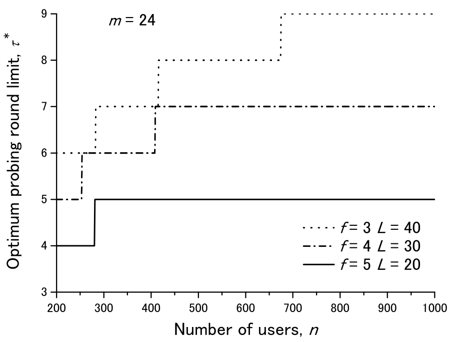
We first show that there does exist an optimum setting of probing round limit at which the per node throughput capacity is maximized. We considered two network scenarios, i.e., $(n = 300, m = 16, L = 30)$ and $(n = 500, m = 24, L = 40)$, and for each network scenario we examined three different settings of bundle redundancy limit f. The corresponding results were summarized in Fig. 2. One can easily observe from Figs. 2a and 2b that, for each setting of f there, there does exist an optimum setting of τ to achieve the maximum μ. For example, for the settings of $f = 2, 3$ and 4 in Fig. 2a, a maximum per node throughput of 1.07×10^{-2}, 1.23×10^{-2} and 1.39×10^{-2} is achieved at the setting $\tau = 5, 6$ and 7, respectively; while for the settings of $f = 3, 4$ and 5 in Fig. 2b, a maximum per node throughput of 9.73×10^{-3}, 1.13×10^{-2} and 1.28×10^{-2} is achieved at the setting $\tau = 8, 9$ and 9, respectively.

Hereafter, we denote by μ^* the maximum per node throughput capacity and by τ^* the corresponding optimum setting of probing round limit, and proceed to explore their relationship with the control parameters, such as the number of mini-slots L, the bundle redundancy limit f and the number of users n.

Fig. 3 shows how the τ^* varies with the number of mini-slots L and the bundle redundancy limit f. As shown in Fig. 3a, τ^* monotonically increases as L varies from 10 to 50 and τ^* is actually a piecewise function of L, i.e., a specific value of τ^* can only apply to a small range of L. A further careful observation of Fig. 3a indicates that under the same setting of L, the τ^* value of a bigger f is also bigger. It can be interpreted as follows: by adopting a bigger f (i.e., allowing more relay nodes to carry redundant copies for a bundle), the probability that the source can find a relay node without carrying a copy in a single probing becomes smaller, therefore, the source needs to conduct more rounds of probing



(a) μ^* vs. n.



(b) τ^* vs. n.

Fig. 4. Maximum throughput capacity μ^* and the optimum probing round limit τ^* vs. the number of users n.

so as to identify an eligible receiver and deliver out a new copy.

For the case of $(n = 450, m = 24)$ and $L = \{20, 30, 40\}$, the relationship between τ^* and the bundle redundancy limit f is illustrated in Fig. 3b. One can see from Fig. 3b that as f varies from 1 to 10, for each setting of L there, the τ^* first increases and then decreases. Specifically, for the setting of $L = 20, 30$ and 40, the τ^* starts to decrease as f increases beyond $f = 8, 7$ and 6, respectively. Actually, such behavior is due to the limitation of τ_0 in (7). According to (7), one can see that the τ_0 monotonically decreases as f increases up which unavoidably results in a narrower range for the selection of τ, and thus the curves of all three settings in Fig. 3b finally converge to the same one.

With m fixed as $m = 24$, we summarize in Fig. 4 how the number of users n would affect the maximum throughput capacity μ^* and the corresponding τ^* under the settings of $(f = 3, L = 40)$, $(f = 4, L = 30)$ and $(f = 5, L = 20)$. As shown in Fig. 4a that, for each setting of (f, L) there, the μ^* diminishes quickly as n increases up. For example, for the case of $(f = 3, L = 40)$, the μ^* of $n = 600$ is 8.64×10^{-3}, which is almost 0.68 times that of $n = 300$ (1.26×10^{-2}). From Fig. 4b, one can easily see that the corresponding τ^* is actually a piecewise function of n, which is similar to that in Fig. 3a. A further careful observation of Fig. 4b indicates that as n increases up, the optimum setting τ^* becomes gradually insensitive to the variations of n, i.e., a specific value of τ^* can apply to a longer range of n. However, this is not the case for that in Fig. 3a, which means that τ^* depends much more heavily on the variations of L.

V. Conclusion

In this paper, we have investigated the per node throughput capacity of probing-based two-hop relay MANETs. Different from previous works, we developed closed-form throughput capacity results where the time cost taken to probe for an eligible receiver in a time slot was carefully considered in the theoretical analysis. Extensive numerical results were further provided to explore the maximum throughput capacity μ^* and the corresponding optimum probing round limit τ^*. Our results show that μ^* diminishes quickly as the number of users increases, and τ^* is actually a piecewise function of the probing time limit, the redundancy limit and the number of users.

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