On the Optimal Transmission Distance for Power-aware Routing in Ad hoc Networks

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Abstract—Power-aware routing for battery-powered wireless ad hoc networks is crucial to insure the longevity of such networks. Most contemporary research that attempts to minimize the energy consumption does so via short distance transmissions. However, this transmission strategy leads to an increase in the number of network operations, and thus increases the probability of collision, which results in extra energy consumption for retransmissions. In this paper, we show that the minimum transmission distance does not result in the minimum energy consumption, and find the optimal transmission distance such that the energy consumption of the ad hoc network is minimal.

I. INTRODUCTION

Many operations such as disaster relief or surveillance operations are carried out in situations with no infrastructure support. Wireless ad hoc networks, shown in Fig. 1, are a robust solution that allow nodes to organize themselves into a network without the need for infrastructure support. Furthermore, in the absence of infrastructure it is difficult to have centralized Medium Access Control (MAC), therefore decentralized Carrier Sense Multiple Access (CSMA) is more practical to realize. Energy efficiency is very important for battery-powered wireless ad hoc networks. Also, since the share of energy consumption attributed to communications is larger than the computation costs [1], many researchers have investigated power-aware routing for wireless ad hoc networks.

According to the work in [2]–[4], the energy consumption of a single successful transmission, \( e(d_{\text{Transmit}}) \), can be quantified as follows:

\[
e(d_{\text{Transmit}}) = \epsilon_1 d_{\text{Transmit}}^\vartheta + \epsilon_2.
\]

Here, \( e(d_{\text{Transmit}}) \) is proportional to the displacement between the transmitting node and the receiving node, \( d_{\text{Transmit}} \). The parameter \( \vartheta \) is the path loss exponent that is dependent on the wireless fading environment, its value is usually from 2 to 4. The term \( \epsilon_1 \) is a constant specific to the wireless system. \( \epsilon_2 \) is the electronics energy, characterized by factors such as digital coding, modulation, filtering, and spreading of the signal.

Based on only the energy consumed for a successful transmission, most contemporary work on power-aware routing has advocated the use of short distance transmissions\(^1\) [5], [6]. Fig. 1 shows an example of the aforementioned transmission strategy. The transmitting node S wants to transmit a packet to node D. Since the path that goes through the relay node R\(_1\) requires longer transmission distances than the other path, a contemporary power-aware routing scheme opts for the latter path because it uses short transmission distances. However, using short transmission distances increases the number of hops, and also the number of required transmissions. These two factors increase the probability of packet collision, which results in increased energy consumption for retransmissions. Therefore, there is a relationship between the transmission distance and the power consumed to deliver a packet from source to destination, which still remains largely unknown.

The previous works that have investigated the transmission distance that minimizes the energy consumption failed to grasp the above mentioned relationship due to assuming an ad hoc network that is saturated, i.e., where all nodes have an infinite number of packets to transmit, and the probability of transmission depends solely on the Contention Window (CW) parameter of IEEE 802.11. However, it is noticeable that even within the same path from source to destination, the number of nodes that buffer and forward varies significantly with the transmission distance, and that the transmission probability of a node will also accordingly change.

In our paper, we consider a general CSMA with Collision Avoidance (CSMA/CA), where each node has a limited number of packets to transmit and the probability of transmission is closely related to the transmission distance to accurately capture the relationship between the transmission distance and the energy consumed in the network. The reminder of the paper is organized as follows. Sec. II presents a review of related works, followed by Sec. III, which presents our energy consumption model. Sec. IV presents numerical results of our model. We finalize this paper with a conclusion in Sec. V.

\(^1\)The minimum (shortest) transmission distance is determined by the closest relay node to the transmitting node. This changes depending on each node’s position.
Fig. 2. The considered Markov chain model with three states, idle, transmit, and collide, and state transition probabilities, $P_{ii}$, $P_{it}$, and $P_{ic}$.

II. RELATED WORKS

Banerjee and Misra [1] pointed out that formulating the link cost based on only the energy consumption of a single transmission is misleading, and a proper metric should include the cost for necessary retransmissions due to link errors. They propose a power-aware routing cost in which links have a specific error rate. The error rate they use does not have any relationship with the condition of the network, i.e., it does not take into account the relationship between the transmission distance and probability of collision.

Deng et al. [2] analyzed the transmission distance that increases energy efficiency. They define energy efficiency as the ratio between the progress of a transmission and the energy consumption of the transmission. Then, they use this definition of energy efficiency to find the optimal transmission distance. Progress of a transmission is how close the packet that is being transmitted gets to its destination. In their work, the energy consumption of a transmission is that of a single successful transmission, which does not take account of transmission failures.

Gabriel et al. [7] investigated the issue of choosing the optimal transmission distance such that the energy consumption is minimal in an IEEE 802.11 network. They use a collision model composed of two Markov chains to evaluate the effect of collisions on the energy consumption. In their network all nodes have an infinite number of packets to transmit, and the probability of transmission depends solely on the CW parameter of IEEE 802.11. The work of Alawieh and Assi [8] has studied the effect of transmission distance on energy consumption in an IEEE 802.11 network with directional antennas. They use a similar collision model to that of Gabriel et al. [7], and make the same assumptions of infinite amount of packets to send and probability of packet transmission of Gabriel et al. [7]. Both [7], [8] assume that the probability of transmission is independent of the transmission distance. In contrast, our work takes into account the transmission distance when calculating the probability of collision so that the relationship between the transmission distance and the energy consumption can be accurately captured.

III. ENERGY CONSUMPTION MODEL

In this section we derive an analytical model to study the effect of transmission distance, $d_{\text{Transmit}}$, on the total energy consumed to transmit a packet from source to destination, which includes the energy consumed for retransmissions due to collisions. This model will show the $d_{\text{Transmit}}$ that renders the minimum energy consumption of the wireless ad hoc network. To include the energy consumption of retransmissions due to collisions we use the collision model in Sec. III-A. The mean value for traffic per node is given in Sec. III-B, which allows the calculation of the probability of packet arrival in Sec. III-C, which will be used in the collision model. Finally, Sec. III-D gives an expression for energy consumption as a function of $d_{\text{Transmit}}$ and other parameters.

Before going forward with the derivation, we describe our model assumptions, which are unique to this work. For tractability, we assume a uniformly distributed network like many other works [1], [7]. A single channel is spatially shared among nodes in the same area (spatial reuse), in other words, if a pair of nodes are communicating, a collision occurs when one or more node(s) transmit within the transmission distance of either the transmitting or receiving node. Nodes use a 1-persistent access strategy, wherein a node that has a packet to transmit senses the channel. If the channel is sensed free, the packet is transmitted. If the channel is busy, the node monitors the channel and transmits the packet when the channel is sensed idle. The transmission distance is equal for all nodes in the network, which is a very commonly used assumption [2], [8]. All nodes have equal priority to transmit, each node can have at most a single packet to transmit per time slot, and all packets are of the same size. Each node has a finite number of packets to transmit in a round. A round is a specific period of time in which a node transmits all of its packets.

A. Collision model

We model the wireless node’s states by using a three-state Markov chain similar to that of [9], [10], the model is shown

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{ii}$</td>
<td>Transition probability from the idle state to itself</td>
</tr>
<tr>
<td>$P_{it}$</td>
<td>Transition probability from the idle to the transmit state</td>
</tr>
<tr>
<td>$P_{ic}$</td>
<td>Transition probability from the idle to the collide state</td>
</tr>
<tr>
<td>$p$</td>
<td>Probability that a packet arrives to a node</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Node density</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>State transition matrix</td>
</tr>
<tr>
<td>$S$</td>
<td>Steady-state vector</td>
</tr>
<tr>
<td>$\omega_i$</td>
<td>Steady-state probability of idle state</td>
</tr>
<tr>
<td>$\omega_t$</td>
<td>Steady-state probability of transmit state</td>
</tr>
<tr>
<td>$\omega_c$</td>
<td>Steady-state probability of collide state</td>
</tr>
<tr>
<td>$E[h]$</td>
<td>Average hop count</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Packet generation rate</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of nodes in the network</td>
</tr>
<tr>
<td>$T$</td>
<td>Number of time slots per round</td>
</tr>
</tbody>
</table>
in Fig. 2, and it has three states, namely, idle, transmit, and collide. The variables used in our analytical model are listed in Table 1. Furthermore, as shown in Fig. 2, \( P_{ii}, P_{it}, \) and \( P_{ic} \) are the state transition probabilities, which correspond to the probabilities of transmission from idle state to idle, transmit, and collide, states respectively. Their derivation method is similar to that shown in [9]. The probability that no transmission occurs, \( P_{ii} \), can be quantified as follows:

\[
P_{ii} = (1 - p)^{Num\{A\}}. \tag{2}
\]

Here, \( p \) denotes the probability that a packet arrives at a node to be transmitted, and \( Num\{A\} \) is the number of nodes in the area \( A \). \( A \) is the area that is covered by the transmission distance of a single node, shown in Fig. 3. \( Num\{A\} \) can be written as

\[
Num\{A\} = \varphi \pi d_{Transmit}^2. \tag{3}
\]

The probability that a node successfully transmits, \( P_{it} \), occurs when only one node within the areas \( A \) and \( B \), shown in Fig. 3, transmits. It takes the following form:

\[
P_{it} = p(1 - p)^{Num\{A\} + Num\{B\}}. \tag{4}
\]

Here, the number of nodes in area \( B \), \( Num\{B\} \), shown in Fig. 3, can be evaluated according to the following equation [11]:

\[
Num\{B\} = \varphi \pi d_{Transmit}^2 - 2d_{Transmit}^2(\arccos \frac{1}{2} - \frac{1}{2} \sqrt{1 - \frac{1}{4}}). \tag{5}
\]

Since the summation of \( P_{ii} \), \( P_{it} \), and \( P_{ic} \) is equal to one, the probability of collision, \( P_{ic} \), takes the following form:

\[
P_{ic} = 1 - P_{ii} - P_{it}. \tag{6}
\]

The state transmission matrix of the Markov chain shown in Fig. 2, \( \Phi \), may be written as:

\[
\Phi = \begin{pmatrix} P_{ii} & 1 & 1 \\ P_{it} & 0 & 0 \\ P_{ic} & 0 & 0 \end{pmatrix}. \tag{7}
\]

Since all the entries of the above matrix are positive, this matrix is regular and has a steady state. Let \( S \) be the steady state vector of \( \Phi \) whose elements are the steady state probabilities of the Markov chain shown in Fig. 2. \( S \) takes the following form:

\[
S = (\omega_i \omega_t \omega_c)^T, \tag{8}
\]

where \( \omega_i \), \( \omega_t \), and \( \omega_c \) are the steady-state probabilities of the Markov chain shown in Fig. 2 in the idle, transmit, and collide states, respectively. Then, \( S \) is an eigenvector of \( \Phi \) with an eigenvalue \( \lambda = 1 \) [10], therefore,

\[
\Phi S = S \\
(\Phi - I_3)S = 0. \tag{9}
\]

Here, \( I_3 \) is the identity matrix of rank 3. The above equation describes a homogeneous system of linear equations with \( \Theta = (\Phi - I_3) \), where

\[
\Theta = (\Phi - I_3) = \begin{pmatrix} P_{ii} - 1 & 0 & 0 \\ P_{it} & -1 & 0 \\ P_{ic} & 0 & -1 \end{pmatrix}. \tag{10}
\]

Furthermore, the system described in Eq. (9) has many possible solutions, and to get a unique solution an extra condition is required. Since \( S \) is a probability vector, its elements should add up to one, i.e.,

\[
\omega_i + \omega_t + \omega_c = 1. \tag{11}
\]

We exchange any row from Eq. (9) with Eq. (11) to get a unique solution (we choose the first row). Thus, the result is,

\[
\begin{pmatrix} 1 & 1 & 1 \\ P_{it} & -1 & 0 \\ P_{ic} & 0 & -1 \end{pmatrix} \begin{pmatrix} \omega_i \\ \omega_t \\ \omega_c \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}. \tag{12}
\]

The above system’s solution can be easily found with algebraic manipulations. It is found to be

\[
\omega_i = \frac{1}{2 - P_{ii}} \tag{13}
\]
\[
\omega_t = P_{it}\omega_i \tag{14}
\]
\[
\omega_c = P_{ic}\omega_i. \tag{15}
\]

**B. Average traffic per node**

Here, we give an expression for the average amount of traffic flowing through a node. Similar to the analysis of [7], [8], [12], our model assumes that the traffic patterns are uniform, i.e., the source and the destination nodes are randomly chosen in a uniform manner. Let each node generate packets at a rate of \( \sigma \). Consider two nodes, \( i \) and \( j \). Let \( E[h] \) denote the average hop count between any source and any destination, which can be formulated as

\[
E[h] = \frac{d_{s\rightarrow d}}{d_{Transmit}}. \tag{16}
\]

where \( d_{s\rightarrow d} \) is the average displacement between sources and destinations. On average there are \( (E[h] - 1) \) relay nodes between any source and any destination. Node \( i \) may become a relay node for node \( j \) with probability

\[
\frac{(E[h] - 1)}{N - 1}. \tag{17}
\]
Here, \( N \) is the number of nodes in the network. The expected amount of relay traffic arriving at node \( i \) from node \( j \) is

\[
\sigma(E[h] - 1) \times \frac{1}{N - 1}.
\]  

(18)

Since there are \((N-1)\) other nodes in the network, node \( i \) may be a relay node for the other \((N-1)\) nodes with a probability of

\[
(N-1) \times \frac{(E[h] - 1)}{N - 1} = (E[h] - 1).
\]  

(19)

Also, the expected amount of relay traffic for node \( i \) is

\[
\sigma(E[h] - 1).
\]  

(20)

The average traffic per node is equal to the traffic generated by the node itself and the traffic it has to relay, i.e.,

Average traffic = own traffic + relay traffic

\[
= \sigma + \sigma(E[h] - 1) = \sigma E[h]
\]  

(21)

C. Packet arrival

We derive an expression for the probability of packet arrival. Let there be \( T \) time slots per round. If on average there are \( \sigma E[h] \) packets flowing through each node, then the average traffic rate per time slot is

\[
\sigma_T = \frac{\sigma E[h]}{T}.
\]  

(22)

Assuming that the traffic is Bernoulli such as that in [13]–[15], then the probability that a packet arrives, \( p \), can be given as:

\[
p = \sigma_T.
\]  

(23)

D. End-to-end energy consumption

Herein, we derive a formula for the energy consumption attributed to transmitting a packet from a source to destination, \( E_{s->d} \). In general, \( E_{s->d} \) is given as:

\[
E_{s->d} = \text{Average hop count} \times \text{energy consumption per hop}
= E[h] \times (E_{\text{Transmit}} + E_{\text{Collision}}).
\]  

(24)

Here, \( E_{\text{Transmit}} \) and \( E_{\text{Collision}} \) are the energy consumed for a successful transmission and energy consumed for retransmissions due to collisions, respectively. \( E_{\text{Transmit}} \) can be calculated from Eq. (1). \( E_{\text{Collision}} \) can be expressed as:

\[
E_{\text{Collision}} = \{\text{Probability(1st collision)}
+ \text{Probability(2nd collision)}
+ \ldots \text{Probability}(n^{th} \text{collision})\}
\times \text{energy of a collision}.
\]  

(25)

The energy consumption of a collision is equal to that of single successful transmission. Similar to [16], we assume that the probability of collision is independent of the number of previously occurred collisions. Thus, we can rewrite Eq. (25) as:

\[
E_{\text{Collision}} = \sum_{i=1}^{\infty} e(d_{\text{Transmit}}) \omega_c^i = e(d_{\text{Transmit}}) \sum_{i=1}^{\infty} \omega_c^i.
\]  

(26)

Here, since \( \omega_c < 1 \), Eq. (26) converges to:

\[
E_{\text{Collision}} = \frac{\omega_c}{1 - \omega_c} e(d_{\text{Transmit}}).
\]  

(27)

IV. NUMERICAL RESULTS

By using the model derived in Sec. III we show the energy consumption of a uniformly distributed wireless ad hoc network. Table II lists the parameter settings used in this paper. The constants of Eq. (1), \( \epsilon_1 \) and \( \epsilon_2 \), are set according to the values reported in [17]. The path loss exponent, \( \eta \), is set to 2 according to the value adopted in [2], [7]. The average displacement between sources and destinations, \( d_{s->d} \), is set according to the value reported in [7]. The number of time slots per round, \( T \), is set to a relatively high value of 1000 to accommodate all operations in the network. The node density, \( \varphi \), and the packet generation rate, \( \sigma \), are both set to unity for simplicity.

Fig. 4 shows the plot of Eq. (24). The transmission distance, \( d_{\text{Transmit}} \), is varied from 0.5 to 10 m. As can be clearly seen from the graph, the energy consumption for transmitting from source to destination is minimum when the transmission distance is approximately 2.1 m. This point achieves the

![Graph showing energy consumption](image-url)
optimal balance between short and long distance transmissions, such that the summation of the energy consumption per transmission and for retransmissions due to collisions is minimized.

Although the value of the path loss exponent, $\vartheta$, is assumed to be 2 in most works, Eq. (1) indicates that its effect on the energy consumption is nontrivial. Therefore, we explore its effect on the optimal value of transmission distance that yields the minimum $E_{s->d}$. Fig. 5 shows the plot of Eq. (24) for several values of path loss exponent, $\vartheta$. The optimal values of $d_{\text{Transmit}}$ are indicated with vertical lines. The results show that when an environment has a large value of $\vartheta$, the value of $d_{\text{Transmit}}$ that decreases the energy consumption of the wireless ad hoc network is also decreased. The reason behind this is, that the growth of Eq. (1) significantly increases with higher values of $\vartheta$. The optimal values of $d_{\text{Transmit}}$ for different values of $\vartheta$ are listed in Table III.

From the results of our analysis, we conclude that the minimum transmission distance does not result in the minimum energy consumption, and find the transmission distance that does result in the minimum energy consumption.

V. CONCLUSION

In this paper, we have investigated the issue of choosing the optimal transmission distance to minimize the energy consumption of wireless ad hoc networks. While most contemporary research attempts to minimize the energy consumption via short distance transmissions, choosing the minimum transmission distance does not lead to minimum energy consumption. In practice, decreasing the transmission distance increases the number of concurrent transmissions in the network, which increases the probability of collision and thus requires more energy for retransmissions. We show via analytical modeling that the minimum transmission distance does not lead to the minimum energy consumption, and find the optimal transmission distance that results in the minimum energy consumption.

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