Performance Modeling of Three-Hop Relay Routing in Intermittently
Connected Mobile Networks

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Performance Modeling of Three-Hop Relay Routing in Intermittently Connected Mobile Networks

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Abstract—A significant amount of works has been done to model the delivery performances in Intermittently Connected Mobile Networks (ICMNs). However, available works considered either the two-hop relay routing or the epidemic routing, which actually represent two extreme cases of the message delivery process in ICMNs. In this paper, we take one step ahead and focus on the three-hop relay routing where each message travels at most three hops to reach the destination. Under such a scheme, besides that the source can send a message copy to each node it meets, a relay which receives the message directly from the source can also replicate the message to other nodes, while a relay node which receives the message from another relay can only forward the message to the destination. In order to characterize the complicated message delivery process under the three-hop relay routing, a multidimensional Markov chain theoretical framework is developed. Based on the Markov chain framework and block matrix theory, closed-form expressions are further derived for the important message delivery delay and delivery cost. Extensive numerical results are also provided to explore the achievable delivery performances under the three-hop relay.

I. INTRODUCTION

Intermittently Connected Mobile Networks (ICMNs) consisting of sparsely distributed mobile nodes, usually suffer from dramatic topology changes and frequent network partitions [1]–[3]. Consequently, there exists no contemporaneous end-to-end path most of the time and the traditional route based routing schemes, such as Dynamic Source Routing (DSR), Ad hoc On-Demand Distance Vector (AODV), etc., fail to function properly in such intermittently connected environment. Instead, the store-carry-forward kind of routing which receives the message from another relay can only forward the message to the destination. In order to characterize the complicated message delivery process under the three-hop relay routing, a multidimensional Markov chain theoretical framework is developed. Based on the Markov chain framework and block matrix theory, closed-form expressions are further derived for the important message delivery delay and delivery cost. Extensive numerical results are also provided to explore the achievable delivery performances under the three-hop relay.

A significant amount of works has been done to model the delivery performances for store-carry-forward routing schemes in ICMNs. Groenevelt et al. in [4] provided closed-form expressions and also asymptotic approximations for the expected message delivery delay of two-hop relay routing and epidemic routing. Later, Zhang et al. in [5] proposed an Ordinary Differential Equations (ODEs) based model to study the performance of epidemic routing, where quantitative results were obtained for the message delivery delay and the number of message copies sent. More recently, Markov chain based models have also been developed to analyze the performances of two-hop relay routing and its variants [6]–[8].

It is noticed that the two-hop relay routing and the epidemic routing actually represent two extreme cases of the message delivery in the ICMNs. Specifically, in the two-hop relay, a relay node can never replicate the received message to another node except the destination; while in the epidemic routing, a relay node can send out a copy of the received message to any node it meet. In this paper, we take one step ahead and focus on the three-hop relay routing. Under such a routing scheme, besides that the source can send a message copy to each node it meets, a relay which receives the message directly from the source can also replicate the message to other nodes, while a relay node which receives the message from another relay can only forward the message to the destination. To the best of our knowledge, this work represents the first analytical study of three-hop relay routing in literature.

The main contributions of this paper are summarized as follows:

- In Section III, we first develop a multidimensional Markov chain theoretical framework, so as to provide a nice characterization for the complicated message delivery process under the three-hop relay routing.
- Based on the developed Markov chain theoretical framework and the block matrix theory, in Section III we further derive closed-form expressions for the important delivery performances under the three-hop relay, like the expected delivery delay and the expected delivery cost.
- Finally, in Section IV we provide extensive numerical results to further analytically explore the achievable delivery performances under the three-hop relay routing.

II. PRELIMINARIES

A. System Models

We consider a network with n nodes moving in a square area of side length l. The mobile nodes are sparsely distributed in the network area and each node adopts a transmission range \( r \ll l \), so that the network is guaranteed to be disconnected most of the time. There is no pre-existing infrastructure in the network, and for any node pair, they can transmit to each other only when they move into reciprocal transmission range. The channel bandwidth is assumed to be limited and the data that can be successfully transmitted during a contact (or meeting) between any node pair is fixed as \( W \) bits.
Regarding the mobility model, we assume that the nodes move within the square area according to the popular Random Waypoint model (or the Random Direction model) with a scalar velocity of $v$. Similar to [9], [10], we further assume that for any node pair the occurrence of their contacts follows the Poisson distribution. In other words, the inter-meeting times of any two nodes, i.e., the time elapsed between their consecutive contacts, are exponentially distributed with an inter-meeting intensity. Actually, this assumption has been validated in [11] and also widely adopted in literature [4], [12], [13]. According to [11], if we denote by $\lambda$ the inter-meeting intensity between a node pair, then we have

$$\lambda = \frac{8\omega v}{\pi^2},$$  \hspace{1cm} (1)

where the constant $\omega$ is determined as $\omega = 1.368$ (resp. $\omega = 1$) for the Random Waypoint model (resp. for the Random Direction model) [11].

**B. Three-Hop Relay Routing**

Distinguished from previous works [6], [7], [14], we consider in this paper the three-hop relay routing. Under such a routing scheme, besides that the source can send a message copy to each node it meets, a relay which receives the message directly from the source can also replicate the message to other nodes, while a relay which receives the message from another relay can only forward the message to other nodes it meets; while a relay which receives the message directly from the source can also replicate the message to other nodes. Hence, the delivery process of a message $M$ under the three-hop relay routing can be defined by a finite-state absorbing Markov chain.

We use $(A,k)$ to denote an absorbing state that when $D$ receives the message $M$ there are already $k$ relay nodes carrying $M$ in the network, $0 \leq k \leq n-2$. If we further use $(i,j)$ to denote a transient state that there are $i$ tier 1 relay nodes and $j$ tier 2 relay nodes in the network, $0 \leq i \leq n-2$, $0 \leq j \leq n-2$, then one can see that only one of the following transitions illustrated in Fig. 2 may happen in the next time instant.

- SR transition: Source-to-Relay transmission, i.e., $S$ successfully transmits a copy of $M$ to a relay node. As shown in Fig. 2, under such a transition case the number of tier 1 relay nodes will be increased by one and the state $(i,j)$ will accordingly transit to state $(i+1,j)$.

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**III. PERFORMANCE ANALYSIS**

**A. A Markov Chain Model**

Note that with the three-hop relay routing, the message transmission (to a relay node or to the destination $D$) depends only on the current spatial distribution of mobile nodes in the network, i.e., being independent of previous node mobility trajectories. Since the delivery process of a message $M$ ends when the node $D$ receives $M$ and the final number of mobile nodes carrying $M$ in the network is also limited, the delivery process of $M$ under the three-hop relay routing can be defined by a finite-state absorbing Markov chain.
• RR transition: it corresponds to the Relay-to-Relay transmission where a tier 1 relay node successfully replicates $M$ to another node. Obviously, the RR transition will increase the number of tier 2 relay nodes by one and the state $(i,j)$ will accordingly transit to state $(i,j+1)$.

• SD or RD transition: Source-to-Destination or Relay-to-Destination transmission, i.e., $S$ delivers the message $M$ to $D$ or a relay node (tier 1 or tier 2) forwards $M$ to $D$. As shown in Fig. 2, under the SD or RD transition, the state $(i,j)$ will transit into the absorbing state $(A,i+j)$.

• Self-loop transition: no message transmission is conducted and the state $(i,j)$ will transit back to itself, i.e., neither the node $D$ nor a relay node receives the message $M$.

When the network system is in state $(i,j)$, $0 < i < n-2$, $0 \leq j < n-2$, $i+j < n-2$, there are in total $i$ tier 1 relay nodes, $j$ tier 2 relay nodes and $n-2-i-j$ relay nodes without carrying the message $M$. For the transition diagram of state $(i,j)$ defined in Fig. 2, if we denote by $v_0(i,j)$, $v_1(i,j)$, $v_2(i,j)$, and $v_3(i,j)$ the rates of Self-loop transition, SR transition, RR transition, and SD or RD transition, respectively, then we have

$$v_1(i,j) = (n-2-i-j)\lambda,$$

$$v_2(i,j) = (n-2-i-j)i\lambda,$$

$$v_3(i,j) = (i+1)\lambda,$$

and

$$v_0(i,j) = -v_1(i,j) - v_2(i,j) - v_3(i,j) = (i^2 - (n-2-i)j - n + 1)\lambda.$$

It is noticed that for a transient state $(i,j)$ with $i+j = n-2$, since all relay nodes have already received the message $M$, there is no SR transition or RR transition and thus we have $v_1(i,j) = 0$ and $v_2(i,j) = 0$. Similarly, for the transient $(0,0)$ there is no RR transition, i.e., $v_3(0,0) = 0$. Combining with the results in equations (2), (3), (4), and (5), we have that (2), (3), (4), and (5) actually hold for all transient states $(i,j)$, $0 \leq i,j \leq n-2$, $i+j \leq n-2$.

If we integrate the transition diagrams of all transient states, we are able to characterize the message delivery process of three-hop relay routing with an absorbing Continuous-Time Markov Chain (CTMC) as illustrated in Fig. 3, where Figs. 3a, 3b, and 3c each represents a partial case of the full Markov chain. Specifically, Fig. 3a defines the case where there is no more than one tier 2 relay node, i.e., $j = 0$; Fig. 3b represents the case that the tier 1 relay nodes may deliver the message $M$ to at most one more relay given that there are already $j$ tier 2 relay nodes in the network, $1 \leq j \leq n-4$; Fig. 3c shows how the node $D$ may receive $M$ when there are $n-3$ tier 2 relay nodes.

From the CTMC shown in Fig. 3, we can see that there are actually $n-2$ rows of transient states, where the $k_{th}$ row corresponds to the row of transient states $(i,j)$ with $j = k$, $0 \leq k \leq n-3$. If we denote by $L_k$ the number of transient states in the $k_{th}$ row, then we have

$$L_k = \begin{cases} n-1 & \text{if } k = 0, \\ n-2-k & \text{if } 1 \leq k \leq n-3. \end{cases}$$

If we further denote by $\beta$ the total number of transient states in the CTMC, then

$$\beta = \frac{1}{2}(n^2-3n+4).$$

For the convenience of reference, we number these $n-2$ rows of transient states sequentially as $1, 2, \ldots, \beta$ in a left-to-right and top-to-down way. For a transient state $(i,j)$ with sequence number $t$, $t \in [1, \beta]$, without incurring any ambiguity, hereafter we use notations $v_0(t)$, $v_1(t)$, $v_2(t)$, and $v_3(t)$ interchangeably with $v_0(i,j)$, $v_1(i,j)$, $v_2(i,j)$, and $v_3(i,j)$ to denote the rates of Self-loop transition, SR transition, RR transition, and SD or RD transition, respectively. The $n-1$ absorbing states are also labeled sequentially in a similar way such that the absorbing state $(A,k)$ is given a sequence number $k+1$, $k \in [0, n-2]$.

### B. Expected Delivery Delay and Expected Delivery Cost

We first formally define the delivery delay and delivery cost for a message $M$ as follows:

**Delivery Delay:** The delivery delay of a message $M$ is defined as the time elapsed between the time instant when the source $S$ starts to transmit $M$ and the time instant when the destination $D$ receives $M$.

**Delivery Cost:** The delivery cost of a message $M$ is defined as the total number of transmissions it takes $M$ to arrive at the destination $D$. 

![Fig. 3. Transition diagram of the continuous-time Markov chain defined for the three-hop relay routing.](image-url)
We denote by $T_d$ and $C_d$ the delivery delay and delivery cost of message $M$, respectively. Based on the Markov chain framework developed in Section III-A, we are ready to derive closed-form expressions for the expected delivery delay $E\{T_d\}$ and the expected delivery cost $E\{C_d\}$.

According to the Markov chain theory [15], for the Discrete-Time Markov Chain (DTMC) embedded just before the jump times of the CTMC in Fig. 3, the one-step transition matrix $P = (P(i,j))_{(\beta+n-1) \times (\beta+n-1)}$ can be defined as follows:

\[
P = \begin{pmatrix} Q & H \\ 0 & I \end{pmatrix},
\]

where $I$ is the identity matrix of size $(n-1) \times (n-1)$, $Q = (Q(i,j))_{\beta \times \beta}$ defines the one-step transition probabilities among the $\beta$ transient states of the DTMC, and $H = (H(i,j))_{\beta \times (n-1)}$ defines the one-step transition probabilities from the $\beta$ transient states to the $n-1$ absorbing states in the DTMC.

We denote by $N = (N(i,j))_{\beta \times \beta}$ the fundamental matrix of the DTMC, then according to the Markov chain theory [15],

\[
N = (I - Q)^{-1}. \tag{8}
\]

If we further denote by $e$ the initial vector of size $1 \times \beta$ where all entries equal to zero except the first entry being one, i.e., $e = (1, 0, \ldots, 0)$, then we have the following theorem.

**Theorem 1:** The expected message delivery delay $E\{T_d\}$ under the three-hop relay routing can be determined as

\[
E\{T_d\} = -e \cdot N \cdot v_0, \tag{9}
\]

where

\[
v_0 = \left( \frac{1}{v_0(1)}, \frac{1}{v_0(2)}, \ldots, \frac{1}{v_0(\beta)} \right)^T,
\]

with $v_0(t)$ denoting the self-loop transition rate of the $tth$ transient state as defined in (5), $t \in [1, \beta]$.

Next we proceed to derive the expected delivery cost $E\{C_d\}$. Recall that the state $(A, k)$ denotes that when the node $D$ receives the message $M$ there are already $k$ relay nodes each carrying a copy of $M$, $0 \leq k \leq n - 2$. Since it takes one transmission to deliver $M$ to a relay node or $D$, the corresponding delivery cost can be determined as $k + 1$ given that the Markov chain gets absorbed in $(A, k)$. Therefore, we have the following theorem.

**Theorem 2:** The expected message delivery cost $E\{C_d\}$ under the three-hop relay routing can be given by

\[
E\{C_d\} = e \cdot N \cdot H \cdot c, \tag{10}
\]

where $c = (1, 2, 3, \ldots, n-1)^T$.

The proofs for Theorems 1 and 2 are similar to that in [8] and are omitted here due to space limit. Please kindly refer to [8] for details.

Theorems 1 and 2 indicate that in order to derive the $E\{T_d\}$ and $E\{C_d\}$, the only remaining issue is to derive the matrices $N$ and $H$.

### C. Derivations of Matrices $N$ and $H$

According to (8), we can see that in order to derive the matrix $N$ we need to derive the matrix $Q$ first. Note that there are in total $n-2$ rows of transient states and for the $kth$ row of transient states, the transitions happen either among transient states of the $kth$ row or from transient states of the $kth$ row to that of the $(k+1)th$ row, $k \in [0, n-4]$. Therefore, the matrix $Q$ can be defined by the following block matrix.

\[
Q = \begin{bmatrix} Q_0 & Q'_0 \\ Q_1 & Q'_1 \\ \vdots & \vdots \\ Q_k & Q'_k \\ \vdots & \vdots \\ Q_{n-4} & Q'_{n-4} \end{bmatrix} , \tag{11}
\]

where block $Q_k$ of size $L_k \times L_k$ defines the one-step transition probabilities among transient states of the $kth$ row in the DTMC and block $Q'_k$ of size $L_k \times L_{k+1}$ defines the one-step transition probabilities from transient states of the $kth$ row to that of the $(k+1)th$ row in the DTMC, $k \in [0, n-4]$.

Now we proceed to derive the blocks $\{Q_k\}$ and $\{Q'_k\}$.

**Derivation of block $Q_k$:** Let $Q_k(i,j)$ denote the $ij$-entry of block $Q_k$, $i,j \in [1, L_k]$; then the non-zero $ij$-entry of $Q_k$ can be defined as follows.

\[
Q_k(i,i+1) = \frac{v_1(i-a(k),k)}{v_0(i-a(k),k)} , \quad \text{if } 1 \leq i < L_k, \tag{12}
\]

where $a(k) = \begin{cases} 1 & \text{if } k = 0, \\ 0 & \text{if } k \geq 1. \end{cases}$ \tag{13}

When $k = n-3$, $Q_{n-3} = 0$.

**Derivation of block $Q'_k$:** Let $Q'_k(i,j)$ denote the $ij$-entry of block $Q'_k$, $i \in [1, L_k], j \in [1, L_{k+1}]$; then the non-zero $ij$-entry of $Q'_k$ can be defined as follows.

\[
Q'_k(i,i-a(k)) = \frac{v_1(i-a(k),k)}{v_0(i-a(k),k)} , \quad \text{if } 1+a(k) \leq i \leq L_{k-1}. \tag{14}
\]

Based on (8), (11), (12) and (14), the matrix $N$ can be derived accordingly and please refer to [8] for details.

Similarly, the matrix $H$ can be defined as

\[
H = (H_0, H_1, \ldots, H_k, \ldots, H_{n-3})^T, \tag{15}
\]

where the block $H_k$ of size $L_k \times (n-1)$ corresponds to the one-step transition probabilities from transient states in the $kth$ row to the $n-1$ absorbing states in the DTMC, $0 \leq k \leq n-3$.

**Derivation of block $H_k$:** Let $H_k(i,j)$ denotes the $ij$-entry of block $H_k$, $i \in [1, L_k], j \in [1, n-1]$, then the non-zero $ij$-entry of $H_k$ can be defined as follows.

\[
H_k(i,i-a(k)+k+1) = \frac{v_3(i-a(k),k)}{v_0(i-a(k),k)} , \quad \text{if } 1 \leq i \leq L_k. \tag{16}
\]
the delay under a bigger we can see that the performance improvement of delivery interesting to notice from Fig. 4b that the more redundant copies for the message until delivery. It is also A ssho in Fig. 4b, the rate of the source or relay nodes delivering out new message copy increases as one can see that the rate of the source or relay nodes delivering the \( \mathbb{E}\{ C_d \} \) depends only on \( n \) and is actually independent of \( \lambda \). Such independence can be explained as follows: from the absorbing Markov chain in Fig. 3, we can see that the average delivery cost depends only on the final state in which the Markov chain gets absorbed and the corresponding absorbing probability. Furthermore, as shown in equations (2), (3), (4), (5), (12), (14), and (16), all one-step transition probabilities, no matter that from transient states to transient states, or that from transient states to absorbing states, are actually independent of \( \lambda \).

V. CONCLUSION

In this paper, we have investigated the delivery performances of three-hop relay routing in ICMNs. Specifically, we first developed a multidimensional absorbing Markov chain based theoretical framework to characterize the complicated message delivery process under the three-hop relay routing. With the help of the Markov chain framework, we further derived closed-form expressions for both the expected message delivery delay \( \mathbb{E}\{T_d\} \) and the expected message delivery cost \( \mathbb{E}\{ C_d \} \) via the theory of block matrix. Finally, extensive numerical results were provided to illustrate how the \( \mathbb{E}\{T_d\} \) and \( \mathbb{E}\{ C_d \} \) vary with the inter-meeting intensity \( \lambda \) and the number of nodes \( n \). Our results indicate that \( \mathbb{E}\{ C_d \} \) is independent of \( \lambda \) and \( \lambda \) can only affect \( \mathbb{E}\{T_d\} \).

IV. NUMERICAL RESULTS

Based on the theoretical framework developed in Section III, we now proceed to analytically explore the delivery performances \( \mathbb{E}\{T_d\} \) and \( \mathbb{E}\{ C_d \} \) under the three-hop relay routing. With the inter-meeting intensity \( \lambda \) (contacts / hour) fixed as \( \lambda = \{0.121, 0.104, 0.081\} \), we summarize in Fig. 4 how the \( \mathbb{E}\{T_d\} \) and \( \mathbb{E}\{ C_d \} \) vary with the number of nodes \( n \). The \( \mathbb{E}\{T_d\} \) and \( \mathbb{E}\{ C_d \} \) under other settings of \( \lambda \) can also be obtained by our theoretical framework. One can easily observe from Fig. 4a that for all the settings of \( \lambda \) there, the \( \mathbb{E}\{T_d\} \) drastically diminishes with \( n \). For example, for the case \( \lambda = 0.081 \), the \( \mathbb{E}\{T_d\} \) of \( n = 80 \) is 1.032, which is almost 0.40 times that of \( n = 20 \) (2.553). From equations (2) and (3), one can see that the rate of the source or relay nodes delivering out new message copy increases as \( n \) increases up. Therefore, we have a faster message delivery speed and thus a smaller \( \mathbb{E}\{T_d\} \). A further careful observation of Fig. 4a indicates that with the same setting of \( n \), a bigger value of \( \lambda \) can also result in a smaller \( \mathbb{E}\{T_d\} \).

Fig. 4b illustrates the relationship between \( \mathbb{E}\{ C_d \} \) and \( n \). As shown in Fig. 4b, the \( \mathbb{E}\{ C_d \} \) increases almost linearly as \( n \) varies from 20 to 100. Combining with the results in Fig. 4a, we can see that the performance improvement of delivery delay under a bigger \( n \) actually comes with distributing out more redundant copies for the message until delivery. It is also interesting to notice from Fig. 4b that the \( \mathbb{E}\{ C_d \} \) of all three \( \lambda \) settings there coincide with each other, which means that

\[ \mathbb{E}\{ C_d \} \text{ depends only on } n \text{ and is actually independent of } \lambda. \]

\[ \text{Such independence can be explained as follows: from the absorbing Markov chain in Fig. 3, we can see that the average delivery cost depends only on the final state in which the Markov chain gets absorbed and the corresponding absorbing probability. Furthermore, as shown in equations (2), (3), (4), (5), (12), (14), and (16), all one-step transition probabilities, no matter that from transient states to transient states, or that from transient states to absorbing states, are actually independent of } \lambda. \]

\[ \mathbb{E}\{T_d\} \text{ and } \mathbb{E}\{ C_d \} \text{ vary with the inter-meeting intensity } \lambda \text{ and the number of nodes } n. \]

Our results indicate that \( \mathbb{E}\{ C_d \} \) is independent of \( \lambda \) and \( \lambda \) can only affect \( \mathbb{E}\{T_d\} \).

REFERENCES