## On Joint Optimal Placement of Access Points and Partially Overlapping Channel Assignment for Wireless Networks

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# On Joint Optimal Placement of Access Points and Partially Overlapping Channel Assignment for Wireless Networks

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Abstract—The design of a wireless network is often critically affected by issues such as determining the optimal density of Access Points (APs) and the optimal channel assignment by exploiting partially overlapped channels (POCs) for significantly improving the network performance in terms of maximizing the overall network capacity. Contemporary research works have traditionally dealt with these two problems in an isolated manner though they should be considered within the same problem formulation. Furthermore, even though deployment of additional APs can improve the network capacity in case there are a few APs in a given area, the APs cannot be indefinitely added to the wireless network. This means that there is an upper bound to the network capacity maximization with respect to the number of APs. In fact, the network capacity starts to dramatically decrease when the number of deployed APs becomes excessive. This performance decrease can be accredited to the substantial interference among the high number of deployed APs. In order to address this challenge, in this paper, we propose an approach to jointly optimize the number of APs and POCs assignment. Our proposal derives the existence of the optimal density of APs with POCs, and models the POC assignment to the deployed APs from a novel perspective. Computer-based simulations are conducted to demonstrate the effectiveness of our proposal.

### I. INTRODUCTION

The WLAN technology in many real-world applications is designed to provide communication service to areas and still having an increasing popularity due to its acceptance-wide and deployment-easy features. In many situations, WLAN planning is needed. For example, how to make a network design for the disaster area beforehand is critical so that it can be deployed promptly after a disaster (e.g., the earthquake, tsunami wreck out communication). Also, it is one of the promising issues in two-tiered wireless networks [1][2][3].

The crucial issues of determining the optimal density of APs and channel assignment have to be considered in the planning phase to maximize the network capacity as the optimization objective. Though deployment of additional APs can improve the network capacity in case there are a few APs in a given area, APs cannot be indefinitely added to the wireless network due to a limited number of channels (e.g., only 3 orthogonal channels are available in the IEEE 802.11b/g).

On the other hand, POCs, which have been indicated to be able to facilitate interference mitigation and improve the network capacity, are used for the communication between users and APs. There are about 11 POCs in the IEEE 802.11b/g with center frequency separated by about 5 MHz while each channel occupies a spread of about 30 MHz as presented in Fig. 1.

There are some overlapped frequencies among adjacent channels, also known as the channel interference. The channel interference decreases with the channel separation (CS) which describes the extent of the frequency overlap. With sufficient separation (no less than 5 channels in the IEEE 802.11b standards) no interference occurs. Currently, either one or three orthogonal channels (channels 1, 6 and 11) are employed in WLAN networks. Recent work shows that a careful design of POC assignment can often lead to significant improvements in spectrum utilization and network performance [4][5][6][7].

In our work, we are focusing on the combination of the optimal number of APs and POC assignment. In the phase of network planning, the network performance can not be guaranteed to be optimized if the number of APs and POC assignment are optimized individually. The network capacity is decreased under the poor scheme of POC assignment in spite of deploying the optimal number of APs. Likewise, the best scheme of POC assignment can not lead to the maximal network capacity if the number of APs is not optimized.

#### II. RELATED WORK

Several methods for network planning in WLAN can be found in the literature, mostly in terms of candidate positions [8][9]. Given multiple candidate positions for APs, select some positions from them so that the network performance is maximized. This method can not determine the optimal density of APs in the sense that it can be seen as a NP-complete problem. Also, measurement driven design [10][11] performs extensive measurements to study the impact of parameters, which has real statistics but hard to be employed and scaled in the planing phase. Besides, in [12], the authors analysed the relationship between MAC parameters and the density of APs. Furthermore, the afore-mentioned works just took into consideration one single channel or the orthogonal channels.

Recent studies also investigated POC assignment mostly from the viewpoint of graph theory, such as weighted conflict graph [14] and directed graph [15]. Unfortunately, these methods usually provided the way to assign channels to maximize



Fig. 1: Frequency spread of various channels in the IEEE 802.11b/g standard [13]. The number 1, 2, ..., 11 are channel indices. There is some overlapping area between two channels nearby, which is called channel interference.

the network capacity, without consideration of the optimal density of APs in the phase of network planning.

In [6], researchers formulated the problem of POC assignment into (0-1) optimization to study how much improvement from POCs compared with using the orthogonal channels. However, it just gave the necessary condition of optimization. It also studied the density of APs using POCs and concluded that POCs can improve the network performance in the high density of APs, without indication of the optimal number of APs. Moreover, it is impossible to apply the approach in the network planning in practice because the proposal considered irregular deployments (e.g., the random deployment).

The rest of the paper is structured as follows. In Section III, it gives the network assumptions, analysis of the existence of the optimal density of APs, proposed scheme of POC assignment and the algorithm to solve the problem. Next in Section IV, the result of the performance evaluation by simulation is discussed. At last, Section V concludes this paper.

### III. OPTIMAL DENSITY OF APS AND POC ASSIGNMENT

In this section, we first make some reasonable assumptions to simplify the model. The primary objective of network planning is to provide the greatest capacity at the worst case, under which it results in the severest interference in the given area. Both the Shannon Capacity and the POC assignment are considered in the analytical studies. From the investigation above, the algorithm is designed to solve the problem.

#### A. Assumptions

We assume the following conditions concerning the interference at the worst case in the network. We first use the interference model in literature [5] to measure the channel interference degree among POCs. For example, the channel interference F for different channel separation CS is depicted in the Table I. Later we propose a new interference model due to the hardness of F and CS based methods to assign POCs.

In addition, because there are multiple channels available and a number of schemes of POC assignment, we also generalize the efficiency for the best scheme of POC assignment and use a scaling factor k that the interference in the optimal TABLE I: Channel Interference [5] F: CS is the separation of two channels  $CS = |c_i - c_j|$ ,  $c_i, c_j$  are channel indices.

F 1 0.96 0.77 0.62 0.32 0 0 0 0 0 0	CS	5	0	1	2	3	4	5	6	7	8	9	10
	F		1	0.96	0.77	0.62	0.32	0	0	0	0	0	0



Fig. 2: Hexagon deployment and the worst case for SINR: users at the edge of coverage of  $AP_j$  interfere with  $AP_i$ ;  $d_{ij}$ : the distance when the severest interference occurs from the users associated to  $AP_i$  to  $AP_j$ .

POC assignment is k times less than that in the random POC assignment.

Furthermore, since it is impossible to determine the optimal density in irregular deployment (e.g., random deployment), density optimization in the regular deployment is studied. The definition of the regular deployment is that there only exists one result of AP placement for the deployment given the number of APs. For the string deployment, we can deploy 1, 2 to n APs uniformly in the area. Also, for the gird deployment we can only determine to deploy 1, 4 to  $n^2$  APs, each of which covers the same size of the area. It is hard to calculate the network capacity because there are numerous placement methods for other numbers of APs. For the calculable capacity are (e.g., 1, 7 and 19 APs) shown in Fig. 2. Though exact regular deployments do not exist in the real scenario, it can be used in the phase of network planning.

We study the network performance in these three regular deployments, which usually are applied in the network planning in practice. On the other hand, the users at the edge of the coverage area of its associated APs lead to the severest interference to others APs [16], which is the worst case to calculate SINR. Additionally, we assume the area is large enough so that the border effect can be ignored. The probability of being interfered by users associated to other basic service set (BSSs, a set of all stations that communicate with the same AP) is the same.

### B. Aggregate Capacity at the Worst Case

Given the deployment of APs, we can calculate the received power  $P_{ij}$  at the worst case when using the same channel in two-ray ground propagation model [17] as follows:

$$P_{ij} = \frac{P_t * g_t * g_r * h_t^2 * g_r^2}{d_{ij}^{\alpha}} = Q * d_{ij}^{-\alpha},$$
(1)

where  $P_t$  is the transmitting power;  $g_t$ ,  $g_r$ ,  $h_t$ ,  $g_r$  are constant system values which are measured by Q as a whole;  $d_{ij}$  is the distance to calculate the interference at the worst case;  $\alpha$  is the path loss exponent and typically  $2 \le \alpha \le 4$ . Thus, the interference  $H_{ij}$  of  $AP_i$  from the users associated to  $AP_j$  using POCs is:

$$H_{ij} = F(c_i, c_j) P_{ij}, \tag{2}$$

where  $c_i$  and  $c_j$  are channels at  $AP_i$  and  $AP_j$ , respectively;  $F(c_i, c_j)$  is channel interference as described in the Table I. SINR is equal to

$$SINR_i = \frac{P_r}{P_0 + \sum_{j \neq i}^n H_{ij}},\tag{3}$$

where  $P_r$  is the received power of  $AP_i$  from its users;  $P_0$  is the ambient noise power. Using Shannon Capacity formula, the achievable aggregate capacity is:

$$C(n) = B * \sum_{i}^{n} log(1 + SINR_i), \qquad (4)$$

where B is the channel bandwidth, about 30MHz in the IEEE 802.11 b/g.

In order to maximize the network capacity, the optimal number of APs is:

$$N_{opt} = \arg\max_{N} C(N) \tag{5}$$

Given the deployment area, when N is small C(N) increases along with N that the increase in capacity dominates that in interference. When N is large, it results in substantial interference due to a limited number of channels such that the increase in interference dominates that in capacity. Thus there is an optimal value of N to maximize the network capacity. We characterize the afore-mentioned performance as the following proposition.

**Proposition 1.** The optimal numbers of APs in the regular deployments, string, grid and hexagon deployments, have the following characteristics:

$$C(N_{opt} - 1) \le C(N_{opt}) > C(N_{opt} + 1),$$
 (6)

*Proof:* Consider the hexagon deployment. Denote the cumulative interference of  $AP_i$  by  $W_i = \sum_j^N H_{ij}$ . In the regular deployments, the potential BSSs interfering with  $AP_i$  can be divided into levels,  $G_1, G_2, ..., G_{\lceil r/2R \rceil}$ , where r is the interference range, R is the radius of coverage area of each AP. The distance to  $AP_i$  for APs in  $G_j$  is the same. The form of  $W_i$  can be written

$$W_{i} = \sum_{j \in G_{1}} F(c_{i}, c_{j}) P_{ij} + \sum_{j \in G_{2}} F(c_{i}, c_{j}) P_{ij} + \dots + \sum_{j \in G_{\lceil r/2R \rceil}} F(c_{i}, c_{j}) P_{ij}.$$
(7)

In the hexagon deployment, the worst case of the interference is from users at the edge of BSSs. The distances between the interfering users in  $G_1, G_2, ..., G_{\lceil r/2R \rceil}$  and  $AP_i$  are R, 3\*R and  $(2*\lceil r/2R\rceil - 1)$ , respectively, which is different from other deployments. According to (1):

$$W_{i} = QR^{-\alpha} \left( \sum_{j \in G_{1}} F(c_{i}, c_{j}) + 3^{-\alpha} \sum_{j \in G_{2}} F(c_{i}, c_{j}) + \dots + (2\lceil r/2R \rceil - 1)^{-\alpha} \sum_{j \in G_{\lceil r/2R \rceil}} F(c_{i}, c_{j}) \right).$$
(8)

Since  $\lceil r/2R \rceil \ge 2$  and the interference is mainly dominated by the closest three levels of interfering BSSs,  $G_1, G_2$  and  $G_3$ ,

$$W_{i} = QR^{-\alpha} \left( \sum_{j \in G_{1}} F(c_{i}, c_{j}) + 3^{-\alpha} \sum_{j \in G_{2}} F(c_{i}, c_{j}) + 5^{-\alpha} \sum_{j \in G_{3}} F(c_{i}, c_{j}) \right).$$
(9)

Given the scheme of POC assignment, assume the interference in the best scheme is k times less than that in the random assignment,

$$W_{i} = QR^{-\alpha}k^{-1} \left( \sum_{i \in G_{1}} \frac{\sum_{c_{j}}^{M} F(c_{i}, c_{j})}{M} + 3^{-\alpha} \sum_{j \in G_{2}} \frac{\sum_{c_{j}}^{M} F(c_{i}, c_{j})}{M} + 5^{-\alpha} \sum_{j \in G_{3}} \frac{\sum_{c_{j}}^{M} F(c_{i}, c_{j})}{M} \right),$$
(10)

where M is the number of POCs. In the hexagon deployment, there are 6, 12 and 18 BSSs in  $G_1$ ,  $G_2$  and  $G_3$ , respectively.

$$W_i = QR^{-\alpha}k^{-1}T(6+3^{-\alpha}*12+5^{-\alpha}*18), \qquad (11)$$

where  $T = \frac{\sum_{c_j}^{M} F(c_i, c_j)}{M}$ . Suppose the coverage area of each AP is equal to  $\pi R^2$  with the radius of R. By using simple geometric arguments and an auxiliary variable R, the aggregate capacity in (4) can be written

$$C(R) = \frac{S}{\pi R^2} B * \sum_{i}^{\frac{S}{\pi R^2}} log(1 + SINR_i)$$

$$= \frac{S}{\pi R^2} B * \sum_{i}^{\frac{S}{\pi R^2}} log(1 + \frac{P_r}{P_0 + W_i}).$$
(12)

Given the exact values for the afore-mentioned parameters as evaluated in Section IV, there exists only one value of  $R = R_{opt}$  corresponding to  $N_{opt}$  so that the derivative of the aggregate capacity C'(R) = 0 and  $C(R_{opt})$  is maximized. The procedure can also be applied to other two deployments, similarly.

### C. POC Assignment in Regular Deployment

We have already introduced the existence of the optimal number of APs to maximize the aggregate capacity. This basically states that the optimal number of APs is at the point of C'(R) = 0, however, without indication of POC assignment.

Specifically, given a poor scheme of POC assignment, it results in low network capacity even though the optimal number of APs is given. The maximal network capacity is obtained through the combination of the optimal number of APs and the best POC assignment.

Usually, it's NP-complete problem for POC assignment due to integer assignment of channel index in terms of interference model F and CS in subsection III-A, mostly from the viewpoint of graph theory [14] and partly from the viewpoint of game theory [18]. In this section, a novel POC assignment is developed and used in the combination of the optimal number of APs in the next subsection.

There is a fixed standard channel center frequency  $f_s^i$  corresponding to the channel *i* as shown in Fig. 1. The POC assignment is identical with the selection of channel center frequency. The basic idea is, firstly, to relax the channel center frequency from the standard channel center frequency to arbitrary frequency so that the optimal selection of channel center frequency is obtained. Then place the constraint on the result to assign POCs. In this paper, we simplify the channel power distribution as a rectangle. The model of channel interference degree is, then, formulated as follows.

$$F(f_i, f_j) = \begin{cases} 1 - \frac{|f_i - f_j|}{B}, |f_i - f_j| < B\\ 0, & otherwise. \end{cases}$$
(13)

where  $f_i$  is the channel center frequency in the range of  $[f_{low}, f_{up}]$ ; *B* is the channel bandwidth. If channel center frequencies of two channels are within interfering separation, their channel interference is  $(1 - \frac{|f_i - f_j|}{B})$ ; otherwise, it is 0. The selection problem of channel center frequency to minimize the interference can be formulated as follows with the new model of channel interference.

min. 
$$\sum_{i=j=i+1}^{n-1} \sum_{j=i+1}^{n} H_{ij}$$
 (14)

s.t. 
$$H_{ij} = F(f_i, f_j) P_{ij}, \forall i, j$$
 (15)

$$F(f_i, f_j) = \begin{cases} 1 - \frac{|J_i - J_j|}{B}, |f_i - f_j| < B\\ 0, \quad otherwise. \end{cases}, \forall i, j \quad (16)$$

$$f_{low} \le f_i \le f_{up}, \forall i. \tag{17}$$

The optimization problem can be solved as a Mixed Integer Linear Problem (MILP) as follows.

min. 
$$\sum_{i}^{n-1} \sum_{j=i+1}^{n} F(f_i, f_j) * P_{ij}$$
 (18)

$$s.t. \quad f_i - f_j + A * y_{ij}^1 \ge 0, \forall i, j \le n$$

$$(19)$$

$$B - (f_i - f_j) + A * y_{ij}^1 \ge 0, \forall i, j$$

$$f_i - f_i$$
(20)

$$F(f_i, f_j) - (1 - \frac{j_i - j_j}{B}) + A * y_{ij}^1 \ge 0, \forall i, j \le n \quad (21)$$

$$f_j = f_j + B + A * y_j^2 \ge 0, \forall i, j \le n \quad (22)$$

$$\begin{aligned} & f_i - f_j + B + A * y_{ij} \ge 0, \forall i, j \le n \\ & - (f_i - f_j) + A * y_{ij}^2 \ge 0, \forall i, j \le n \end{aligned}$$
(23)

$$F(f_i, f_j) - (1 + \frac{f_i - f_j}{B}) + A * y_{ij}^2 \ge 0, \forall i, j \le n \quad (24)$$

$$f_i - f_j - B + A * y_{ij}^3 \ge 0, \forall i, j \le n$$
 (25)

$$F(f_i, f_j) + A * y_{ij}^3 \ge 0, \forall i, j \le n$$

$$(26)$$

$$-B - (f_i - f_j) + A * y_{ij}^4 \ge 0, \forall i, j \le n$$
(27)

$$F(f_i, f_j) + A * y_{ij}^4 \ge 0, \forall i, j \le n$$

$$(28)$$

$$y_{ij}^1 + y_{ij}^2 + y_{ij}^3 + y_{ij}^4 = 3, \forall i, j \le n$$
<sup>(29)</sup>

$$y_{ij}^k = 0, 1, \forall i, j \le n, k = 1, 2, 3, 4 \tag{30}$$

$$f_{low} \le f_i \le f_{up}, \forall i \le n,\tag{31}$$

where A is a constant, which value should be large enough. By introducing (0-1) variables  $y_{ij}^k$ , we reduce the problem to MILP. Constraints (19)-(21), (22)-(24), (25)-(26) and (27)-(28) describe cases  $0 \le f_i - f_j < B$ ,  $-B \le f_i - f_j <$  $0, B \le f_i - f_j$  and  $f_i - f_j < -B$  in (16), respectively. There are multiple (0-1) variables in MILP formulation, which is not efficient to solve when the number of APs becomes large. Since the interference mainly is from BSSs nearby, the complexity of the problem can be reduced by associating BSSs in just the closest three levels of interfering BSSs, for example.

Denote  $f^* = (f_1, f_2, ..., f_n)$  as the solution of channel center frequency assignment from MILP. Place the constraint of standard channel center frequency  $f_s^k$  on  $f^*$  to assign POCs. Since there exists  $k, f_s^k \leq f_i \leq f_s^{k+1}$ , the most simple method is to choose POCs from  $\{c_k, c_{k+1}\}$  corresponding to channel frequencies  $\{f_k, f_{k+1}\}$  for each AP.

Let  $c^*$  be the result of POC assignment in the aforementioned proposal,  $c_{opt}$  the optimal POC assignment, which usually is impossible to obtain in the conventional approaches. It results in deviation when placing the constraint on the result of MILP formulation. The definition of deviation is how far it is from the channel frequency corresponding to  $c^*$ obtained in the proposal to that in the optimal POC assignment corresponding to  $c_{opt}$ . The worst case of the solution in the proposal is  $\exists k, f_i = f_s^k + \frac{B}{2}, \forall i$  and the corresponding POC assignment of  $AP_i$  with standard channel center frequency  $f_s^k$ or  $f_s^{k+1}$ . So the deviation  $dev = \frac{|c^* - c_{opt}|}{B} \leq \frac{|f_s^k - f_i|}{B} = \frac{1}{2}$ . The main merit of our proposal is that it can be regarded as the reference with the minimal interference compared with other proposals since the interference in solution  $f^*$  is not worse than that in the optimal channel assignment.

### D. Algorithm for Determining Optimal Number of APs Using POCs

Now, we propose an algorithm for determining the optimal number of APs and POC assignment based on the aforementioned analysis and MILP formulation.

In order to cover the entire area, the minimal number of APs is given in the line 1. S is the size of the area;  $S_0$  the maximal coverage area of AP;  $g_i$  is the number of APs in the regular deployment. In the string deployment, for example,  $(g_1, g_2, g_3, ...) = (1, 2, 3...)$ ; for the grid deployment,  $(g_1, g_2, g_3, ...) = (1, 4, 9...)$  and  $(g_1, g_2, g_3, ...) = (1, 7, 19...)$ in the hexagon deployment as described in subsection III-A.

As investigated in subsection III-B, the optimal number of APs can be found according to the characteristics in

### Algorithm 1

1: Init number of APs:  $N = argmin_{g_i}(g_i - \frac{S}{S_0}), g_i \ge \frac{S}{S_0}$ ; 2: **loop** solve MILP formulation given the number of AP N; 3: 4: if  $C(g_{i-1}) \le C(N) > C(g_{i+1})$  then N is the optimal number of APs; 5: break; 6: 7: else i = i + 1;8:  $N = g_i;$ 9: 10: end if 11: end loop 12: Place the constraint to assign POCs.

proposition 1. The steps from line 2 to 11 in the algorithm explore the characteristics to find the optimal number of APs while locating the optimal channel center frequency in line 3. The simplest searching method, an additional one every other iteration, is applied. However, other searching techniques, binary search as an example, can also be used to accelerate the algorithm. Once the optimal number of APs is obtained with the optimal channel center frequency, the algorithm places the necessary constraint of the standard channel center frequency to assign POCs to these APs.

### IV. PERFORMANCE EVALUATION

### A. Environment

In this section, we evaluate our proposal presented in Section III. String and grid deployments are used in the area of 1000 meters and (1000 \* 1000) square meters, respectively. The ambient noise power is about -100dbm as common. The path loss exponent  $\alpha$  is 3. The communication range can be calculated about at 270meters for the rate of 11Mbps. The parameter of two-ray ground model Q is 1. The channel bandwidth is about 27.67MHz. The channel center frequency is in the range of [2413.8, 2469.8] MHz. We use Lingo [19] to solve the MILP formulation.

### B. Case Study of String Deployment

On the simple string deployment, we found the optimal number of APs and their POC assignment to maximize the network capacity. The performance of normalized capacity is depicted in Fig. 3. The minimal number of APs to cover the entire area is 3. The network capacity increases along with the number of APs until reaching the optimal number of APs 9. The increase in capacity dominates that in interference. Then it decreases sharply until to the minimal normalized capacity about 3, since there are 3 orthogonal channels in the frequency band of the IEEE 802.11 b/g. The increase in interference dominates that in capacity.

We also give the selection of the optimal channel center frequency calculated in MILP formulation and the result of POC assignment for the string deployment. The results of the varying numbers of APs from 3 to 5 are given as shown in Table. II.



Fig. 3: Simulation result in the string deployment: the optimal number of APs is 9.

TABLE II: Selection of channel center frequency and POC assignment.

#AP	<b>Result in MILP Formulation</b>	POC Assignment
3	2413.8, 2441.8, 2469.8	1, 6, 11
4	2414.5, 2442.1, 2469.8, 2413.8	1, 6, 11, 1
5	2442.1, 2469.8, 2413.8, 2441.5, 2469.2	6, 11, 1, 6, 11

The second column of the table is the result in MILP formulation when deploying APs in a line and the corresponding POC assignment is in the last column. From the table, the POC assignment only lies in the orthogonal channels, that are channel 1, 6 and 11, irrelevant to POCs, which consists with the feature of the string deployment.

The interference  $H_{ij}$  is proportional to the  $F(f_i, f_j) * d_{ij}^{-\alpha}$ , where  $F(f_i, f_j)$  is a linear function of  $|f_i - f_j|$  and  $d_{ij}^{-\alpha}$  is a power function of distance  $d_{ij}$ . Note that APs are placed in a line so that the interference mainly is decided by the  $d_{ij}$ . For  $AP_i$  in order to minimize the interference from  $BSS_i, \forall j \neq i$ , reduce interference from its two neighbours at both the left and the right as much as possible. Thus just assign orthogonal channels to neighbours. If the value of path loss exponent is not greater than 1, POC assignment will arise in the string deployment. Since there are more APs around for the grid or hexagon deployments, the interference is not mainly determined by their neighbours and POCs are applied as illustrated in subsection IV-C. However, if  $\alpha$  is large enough, about  $\alpha \geq 3.5$ , the same result as the string deployment also appears. In addition, the deviation of our proposal can be easily obtained and it is consistent with our deviation analysis. Intuitively, it is also reasonable that the capacity increment drops sharply for the big n in the figure since interference increases quickly in the high density of APs.

### C. Case Study of Grid Deployment

In this subsection, we obtain the result of the grid deployment. In Fig. 4, we plot the normalized capacity for varying numbers of APs deployed in the given area. Similarly, the aggregate capacity increases initially until reaching the optimal number of APs, that is 36 in the grid deployment. Then the capacity decreases sharply due to much interference among BSSs. Note that the utilization of APs in the grid deployment,



Fig. 4: Simulation result in the grid deployment: the optimal number of APs is 36.

4	9	4	9	1	5
11	1	11	1	11	1
6	11	1	6	1	8
11	3	8	1	11	3
7	1	11	6	1	8
2	11	6	1	11	3

Fig. 5: POC assignment in the grid deployment when there is the optimal number of APs. The red square represents AP and the number at the top of red rectangles is the corresponding channel index.

<u>number of APs</u> normalized capacity, is lower than that in the string deployment when the density of APs is large because there are more neighbours and more interference in the grid deployment. The afore-mentioned reason also leads to the full usage of POCs in Fig. 5 as discussed in subsection IV-B. The performance of the hexagon deployment is similar with the grid deployment.

### V. CONCLUSION

In this paper, we addressed the critical issue of combining the density of APs and the assignment of POCs that aims at maximizing the overall network capacity. There is certainly a maximal network capacity by optimizing the combination issue, but may not occur in individual optimization scenarios. In particular, we analyzed the characteristics of the problem, which suggests a relationship between the network capacity and the number of APs in the context of POC assignment. Roughly speaking, additional APs can be deployed to increase the network capacity until reaching the optimal number of APs. However, the network capacity decreases with an excessive number of APs due to the substantial interference among the APs.

Based on our conducted analysis, we also propose an algorithm based on a novel scheme of POC assignment. We developed the model of POC assignment from the perspective of frequency distribution, which explores the interference in depth. Through both analysis and simulation, we demonstrated the effectiveness of our proposal in string, grid and hexagon deployments.

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