A Stochastic Geometry Analysis of D2D Overlaying Multi-Channel Downlink Cellular Networks

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***Abstract*—Based on the tool of stochastic geometry, we present in this paper a framework for analyzing the coverage probability and ergodic rate in a D2D overlaying multi-channel downlink cellular network. Different from previous works, 1) we consider a flexible new scheme for mobile UEs to select operation mode individually, under which a mobile UE decides to establish a cellular link (with a BS) or a D2D link (with a neighboring UE) based on the pilot signal strength received from its nearest BS; 2) we allow a mobile UE which is located far from BSs to connect to a nearby BS via another intermediate UE in a two-hop manner. Our results indicate that the developed framework is very helpful for network designers to efficiently determine the optimal network parameters at which the optimum system performance can be achieved. Furthermore, as corroborated by extensive numerical results, enabling the D2D link based two-hop connection can significantly improve the network coverage performance, especially for the low SIR regime.**

***Index Terms*—Device-to-device communication; downlink; performance analysis; multi-channel.**

I. Introduction

The last decade has witnessed a tremendous increase in the number of mobile devices and also a sharp rise in the demand for mobile data communication [1]. Facing such massive consumer demand for mobile data communications, especially that from the skyrocketing number of mobile user equipments (UEs) and smartphones, we have, however, only rather limited wireless band resources. Obviously, relying solely on the traditional cellular networking cannot meet the great demand. To address this big challenge, a lot of techniques have been proposed in the last decade, such as cognitive radio [2], Femtocells [3], white space [4], device-to-device (D2D) communications [5], etc.

Recently, plenty of ink has been poured on the term of D2D communication, as believed to be one of the most promising alternatives to fulfill the sharply rising consumer demand for wireless data communication [6], [7], in terms of increasing network capacity, improving system throughput and spectral efficiency, extending network coverage and the battery lifetime of UEs, etc. Basically, D2D communication can use either licensed spectrum resources or other unlicensed or unused band [5], [6], [8], [9]. When exploiting cellular band for D2D communication, a D2D pair could use a resource pool for both upstream and downstream, or dedicated resources for upstream/downstream. For the case of D2D pairs and cellular UEs sharing the same spectrum resources (underlay mode), one major advantage is that it is able to achieve the best spectrum efficiency. While for the case of allocating dedicated frequency resources for D2D communications (overlay mode), there is no interference issue between D2D and cellular communications.

There has been a lot of works studying resource allocation and mode selection for D2D communications (see [10] for a detailed survey). Yu *et al.* in [11] explored the problem of resource sharing in D2D communication underlaying cellular network under three different resource-sharing modes. The target was to maximize the throughput of a cellular network consisting of a base station (BS), a cellular UE, and a D2D UE pair, while fulfilling prioritized cellular service constraints. Belleschi *et al.* in [12] also explored the resource sharing of D2D underlaying cellular networks, and proposed a distributed suboptimal joint mode selection and resource allocation scheme to minimize the overall transmission power for all mobile UEs. In [13], Yu *et al.* applied Han-Kobayashi rate splitting based resource sharing scheme for a single cell scenario with a D2D pair and a cellular UE. For the single cell scenario, the optimality of distinct resource allocation modes in terms of sum rate has been discussed under various practical constraints, e.g., minimum and maximum spectrum efficiency [14], maximum transmit power and energy limitation [11], average CSI [15], cellular rate guarantee in SINR domain [16]. Liu *et al.* [17] discussed the mode selection of a single D2D pair between underlay mode and overlay mode by introducing specific relay nodes. Chien *et al.* [18] considered joint mode selection and resource allocation for a more general scenario involving multiple D2D pairs and multiple cellular UEs, where the target is to optimally determine the operation mode, radio resources, and transmit power for sum rate maximization.

Available works, although being able to provide precious insights into resource allocation and mode selection for D2D communications, has one common limitation: they considered a very limited number of BSs and mobile UEs (in most cases, only a single BS with a cellular UE and a D2D pair), which necessarily failed to take into account the spatial distribution of other BSs and mobile UEs and thus the impact of accumulated interference from surrounding cells. Another common limitation is that almost all available D2D researches considered only direct D2D communications. According to the results reported in [19], the opportunity of direct D2D

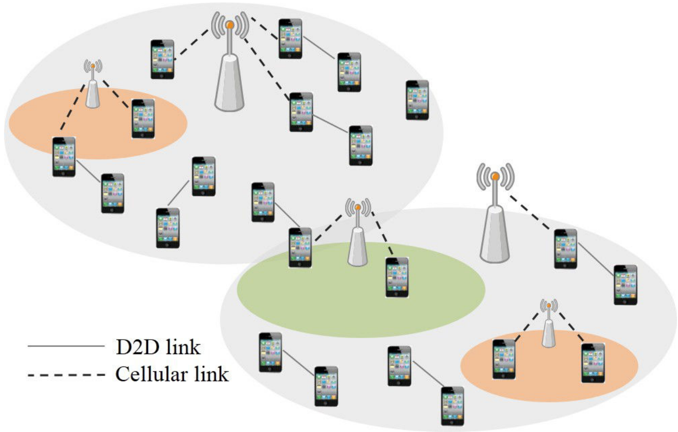


Fig. 1. A cellular network with at most two-hop connection between each mobile UE and a BS, where the solid line denotes the D2D link and the dash line denotes the cellular link.

communication is subject to the requirement that the receiver of the outgoing data and the data holder should be in close proximity, and it cannot detour traffic from congested BSs to adjacent lightly loaded BSs due to its poor flexibility. Towards this end, we, with the help of stochastic geometry theory, study a D2D communication overlaying multi-channel cellular network with a careful consideration of the spatial distribution properties of both BSs and mobile UEs. Furthermore, we consider a more practical network scenario where a mobile UE connects a neighboring BS either in a one-hop cellular link, or in a two-hop connection via another intermediate UE, as illustrated in Fig. 1.

The main contributions of this paper are summarized as follows:

- We consider a flexible new scheme for mobile UEs to select operation mode individually, in which a mobile UE decides to establish a cellular link (with a BS) or a D2D link (with a neighboring UE) based on the pilot signal strength received from its nearest BS. It connects to the nearest BS for data reception if and only if the signal strength (power) received from the nearest BS is larger than a specified threshold. Such threshold based mode selection scheme enables network designers to take fully advantage of cellular links and D2D links and thus to achieve the optimum network performances by adjusting the threshold value, as to be illustrated in Section IV.
- Another contribution point is that different from previous works, we allow a mobile UE which is located far from BSs (e.g., the cell edge area) to connect to a nearby BS via another intermediate UE (called D2D relay) in a two-hop manner. In this way, the downlink traffic is first transmitted over the cellular link between the BS and the D2D relay and then over the D2D link between the D2D relay and the UE. As illustrated by the numerical results in Section IV, such two-hop connection which actually combines the disjoint cellular links and D2D links studied intensively in previous works, although is of significant potential to improve the network coverage in downlink transmissions.
- Utilizing the tool of stochastic geometry, we develop an analytical framework for analyzing the network coverage probability with explicit consideration of spatial distributions of BSs and mobile UEs. The framework can be directly applied for both cases of allowing and not allowing D2D communications, as well as for both schemes of random channel assignment and sequential channel assignment. Based on the framework, the ergodic rate of a generic mobile UE is further derived for the above two cases and the above two channel assignment schemes.
- We present extensive numerical results to illustrate the effectiveness of the threshold based mode selection scheme, the combined two-hop connection, as well as our analytical framework. The flexibility and optimality of the threshold based mode selection is corroborated by the numerical results of various scenarios, which means that network designers can utilize our analytical framework to efficiently determine the optimal threshold, the optimal BS (UE) density, and the optimal transmit power of BSs (UEs). Furthermore, as validated by the numerical results, the combined two-hop connection can significantly improve the network coverage performance, especially for the low SIR regime.

The rest of this paper is outlined as follows. In Section II, we introduce the network model and channel model adopted in this paper. Section III presents the main analytical results of this work, in which tractable expressions are derived for coverage probability and ergodic rate of the downlink scenario. We provide extensive numerical results in Section IV and conclude the whole paper in Section V.

II. System Models

A. Network Model

Consider a cellular network consisting of multiple BSs and mobile UEs. We assume a mobile UE either directly connects to its nearest BS, or connects to its nearest neighboring UE that is directly connected to a BS. Therefore, the connection between a mobile UE and its BS is at most two hops. In such a network, a mobile UE either directly downloads (resp. uploads) data from (resp. to) its nearest BS or via its neighboring UE. For simplicity of expression, we call the link between a BS and a UE cellular link, call the link between two UEs D2D link, and call the intermediate mobile UE in a two-hop connection D2D relay. Fig. 1 shows an example of the considered network scenario. In this work, we focus on the downlink performance analysis.

We assume there are only one-tier BSs and leave the scenario of multi-tier BSs as future work. As the practical deployment of BSs in flat urban area can be well characterized by Poisson point process (PPP) [20], we assume the BSs are spatially distributed according to a homogeneous PPP Φ_b of density λ_b, resulting a network with the well-known Poisson-Voronoi (PV) cell tessellation. The mobile UEs are distributed

TABLE I
NOTATION SUMMARY

Notation	Description
Φ_b	A PPP modeling the spatial locations of BSs
λ_b	Deployment density of BSs
Φ_u	A PPP modeling the spatial locations of mobile UEs
λ_u	Spatial density of mobile UEs
p_b	Transmit power of BSs
p_u	Transmit power of mobile UEs
β	Threshold value for a UE to directly connect to its nearest BS
α	Path-loss exponent
$h_b \sim \exp(\mu_b)$	Fading coefficient of cellular link with mean $1/\mu_b$
$h_u \sim \exp(\mu_u)$	Fading coefficient of D2D link with mean $1/\mu_u$
$\mathbf{N}$	Set of channels for data transmission in cellular links
$\mathbf{M}$	Set of channels for D2D communications
η	The minimum SINR required for coverage
σ^2	Additive channel noise power

in the network region according to another independent homogeneous PPP Φ_u of density λ_u. We assume constant transmit power in this paper, and denote by p_b and p_u, respectively, the transmit power of BSs and that of mobile UEs.

We assume a threshold-based connection setup in this work. Specifically, the connection type (one-hop or two-hop) of a mobile UE is selected according to the received pilot signal strength from the nearest BS (i.e., the BS located within the same PV cell as the UE). For a tagged UE, it connects to the nearest BS for data reception (one-hop connection) if and only if the signal strength (power) received from the nearest BS is larger than a threshold value $\beta > 0$. Note that if the value of β is selected to be too small, almost all mobile UEs will directly connect to its nearest BS and the number of D2D links will become rare which necessarily leads to poor spatial reuse and low spectrum efficiency; on the other hand, if the β is too large, only the UEs that are very close to a BS have direct connection and most of the UEs will suffer outage due to the limitation of maximum two-hop connection. Therefore, the value of threshold β should be carefully tuned, as to be illustrated later.

B. Channel Model

We assume the general power law propagation model and Rayleigh fading for both cellular links and D2D links. Specifically, for a transmitter with transmit power p, the signal power received at a receiver with distance r from the transmitter, is denoted as $p \cdot h \cdot r^{-\alpha}$, where h is the exponentially distributed fading coefficient, and α is the path-loss exponent, $\alpha > 2$. We denote by h_b and h_u the fading coefficient of a cellular link and that of a D2D link, respectively, and assume that the fading coefficients are mutually independent. To characterize the different channel conditions between cellular links and D2D links, we assume $h_b \sim \exp(\mu_b)$, $h_u \sim \exp(\mu_u)$. Besides, the channel noise power is additive and of constant value σ^2.

We consider a D2D communication overlaying multi-channel cellular network [21]. The frequency resources are evenly divided into orthogonal channels. We denote by $\mathbf{N}$ the set of channels for data transmission in cellular links, $\mathbf{N} = \{n_1, n_2, \ldots, n_{|\mathbf{N}|}\}$, with $|\mathbf{N}|$ denoting the number of channels. The other set of channels, $\mathbf{M} = \{m_1, m_2, \ldots, m_{|\mathbf{M}|}\}$, is reserved for data transmissions of D2D links. There is no overlapping channels between $\mathbf{N}$ and $\mathbf{M}$.

A BS assigns channels to cellular links until all $|\mathbf{N}|$ channels have been used up. In a D2D link, the D2D relay randomly selects a channel from $\mathbf{M}$ for D2D communication. The mobile UEs are assumed to have two network interfaces, and are able to transmit and receive data simultaneously in two different channels without causing mutual interference. Therefore, for a UE with two-hop connection with a BS, the downlink traffic is first transmitted over the cellular link and then over the D2D link.

Regarding the channel assignment of cellular links at the BSs, we consider in this paper two assignment schemes: random channel assignment and sequential channel assignment. In the former scheme, each BS randomly selects a channel from the remaining channels and assigns it to a cellular link; while in the latter scheme, the channel assignment is based on channel index, i.e., the channel $n_{i+1} \in \mathbf{N}$, $1 \le i \le |\mathbf{N}| - 1$, will be assigned to a cellular link only after channel n_i has been used.

III. COVERAGE PROBABILITY AND ERGODIC RATE

A. Some Basic Results

Let us denote by r_b the distance between a tagged UE and its nearest BS, then we can see that the pdf of r_b can be expressed as

$$f_{r\_b}(x) = 2\pi\lambda_b x e^{-\lambda_b \pi x^2}.$$

Similarly, if denoting by r_u the distance between a tagged UE and its nearest UE, we have

$$f_{r\_u}(x) = 2\pi\lambda_u x e^{-\lambda_u \pi x^2}.$$

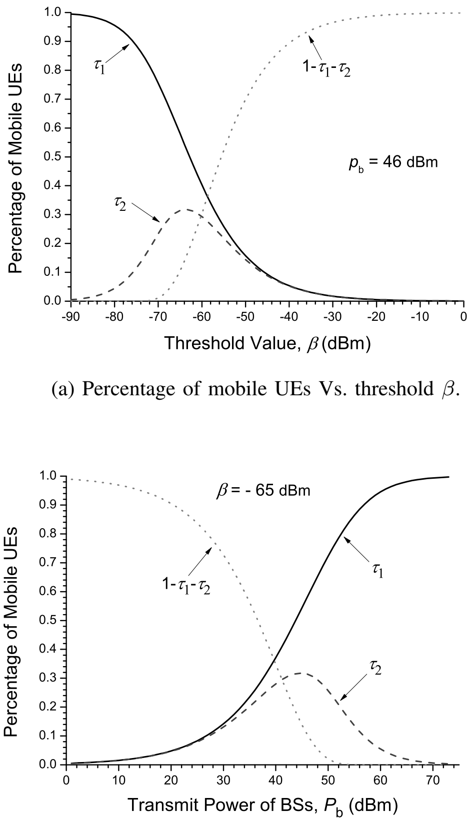
We further denote by τ_1 the probability that the tagged UE has one-hop direct connection to the nearest BS, and denote by τ_2 the probability that the UE has two-hop connection with a BS. The one-hop connection probability

$$\begin{aligned}
\tau_1 &= \Pr(p_b h_b r_b^{-\alpha} > \beta) \\
&= \mathbf{E}_{r_b}\big[\Pr(h_b > \frac{\beta r_b^{\alpha}}{p_b} | r_b)\big] \\
&= \int_0^{\infty} e^{-\mu_b \frac{\beta x^{\alpha}}{p_b}} e^{-\lambda_b \pi x^2} 2\pi\lambda_b x \mathrm{d}x \qquad (1)\\
&= \pi\lambda_b \int_0^{\infty} \exp\Big\{ -\pi\lambda_b v - \frac{\mu_b \beta}{p_b} v^{\frac{\alpha}{2}} \Big\} \mathrm{d}v \qquad (2)
\end{aligned}$$

where (2) follows after substituting $v = x^2$ in (1). From (2), one can see that the UEs which have one-hop cellular connections are of spatial density $\tau_1\lambda_u$. According to the independent thinning property of PPP, the one-hop connected UEs follow a PPP, say Φ_u^1, of density $\tau_1\lambda_u$.

Remark 1: For the special case of $\alpha = 4$,

$$\tau_1 = \frac{\pi\lambda_b}{2}\sqrt{\frac{\pi p_b}{\mu_b \beta}} \exp(\frac{\pi^2 p_b \lambda_b^2}{4\mu_b \beta}) \mathrm{erfc}(\frac{\pi\lambda_b}{2}\sqrt{\frac{p_b}{\mu_b \beta}})$$



(a) Percentage of mobile UEs Vs. threshold β.

(b) Percentage of mobile UEs Vs. transmit power of BSs p_b.

Fig. 2. Illustration of the percentage of mobile UEs which have one-hop cellular connection (denoted by τ_1), two-hop connection via a D2D relay (denoted by τ_2), and no connection due to the limitation of at most two hops between a UE and a BS (denoted by $1-\tau_1-\tau_2$). The other system parameters are configured as $\lambda_b = 1/\text{km}^2$ and $\mu_b = 1$.

where $\text{erfc}(x)$ is the complementary error function, $\text{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-y^2}\text{d}y$.

Since only the one-hop connected UEs are eligible to serve as D2D relays, a UE $u \in \Phi_u \backslash \Phi_u^1$ will have two-hop connection if and only if the nearest neighboring UE $u^* \in \Phi_u^1$ chooses to select u as receiver to form a D2D link. Since the potential receiver u follows another independent PPP of density $(1-\tau_1)\lambda_u$, we can approximately determine the probability of a generic UE having two-hop connection with a BS as ([22], eq.(12))

$$\tau_2 = (1-\tau_1)(1-e^{-\frac{\tau_1}{1-\tau_1}}) \quad (3)$$

From (3), one can see that the mobile UEs that have a two-hop connection with a nearby BS follow a PPP, say Φ_u^2, of density $\tau_2\lambda_u$. For the case that multiple UEs from Φ_u^2 connect to a UE in Φ_u^1, it randomly selects a UE out of them to form a D2D link.

Fig. 2 illustrates how the percentage of mobile UEs which have one-hop cellular connection, two-hop connection, and no connection with a BS, varies with the threshold β and the BS transmit power p_b. One can observe from Fig. 2a that as β increases from $-90\,\text{dBm}$ to $0\,\text{dBm}$, τ_1 (i.e., the percentage of one-hop connected UEs) monotonically decreases while $1-\tau_1-\tau_2$ (i.e., the percentage of UEs with no connection to a BS) monotonically increases. While the behaviors of τ_1 and $1-\tau_1-\tau_2$ in Fig. 2b is just opposite to that in Fig. 2a. Interestingly, in both Fig. 2a and Fig. 2b, the percentage of two-hop connected UEs, i.e., τ_2 first increases and then decreases, reaching a maximum value of 0.32 (resp. 0.32) at around $\beta = -64\,\text{dBm}$ (resp. $p_b = 45\,\text{dBm}$) in Fig. 2a (resp. Fig. 2b), which further validates that the system parameters β and p_b should be carefully tuned so as to optimize the system performance.

After deriving τ_1, now we are ready to analyze the number of one-hop connected UEs within a PV cell. Recall the pdf of a Voronoi cell V can be approximated as

$$f_{pv}(x) \approx \frac{\lambda_b^c c^c x^{c-1} e^{-c\lambda_b x}}{\Gamma(c)}$$

where $c = 3.575$, and $\Gamma(x)$ is the gamma function defined as $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}\text{d}t$.

We denote by $g(k)$ the probability that there are k one-hop connected UEs within a PV cell, $k \geq 0$, then we have

$$\begin{aligned} g(k) &= \int_0^\infty \frac{(\tau_1\lambda_u x)^k e^{-\tau_1\lambda_u x}}{k!} \cdot f_{pv}(x)\text{d}x \\ &= \frac{(c\lambda_b)^c(\tau_1\lambda_u)^k}{(\tau_1\lambda_u + c\lambda_b)^{k+c}} \cdot \frac{\Gamma(k+c)}{\Gamma(c)\Gamma(k+1)}. \end{aligned} \quad (4)$$

Given (4), we proceed to derive the probability of a BS using channel $n_i \in \mathbf{N}$ under the two different channel assignment schemes. We denote by $\xi_r(i)$ the probability that a BS will assign channel n_i for downlink transmission under the random assignment scheme, and by $\xi_s(i)$ the probability that a BS will assign channel n_i under the sequential assignment scheme.

$$\begin{aligned} \xi_r(i) &= \sum_{k=0}^{|\mathbf{N}|} g(k) \cdot \frac{k}{|\mathbf{N}|} + \sum_{k=|\mathbf{N}|+1}^{\infty} g(k) \cdot 1 \\ &= 1 - \sum_{k=0}^{|\mathbf{N}|} g(k)(1 - \frac{k}{|\mathbf{N}|}). \end{aligned} \quad (5)$$

Similarly, $\xi_s(i)$ can be given by

$$\xi_s(i) = 1 - \sum_{k=0}^{i-1} g(k). \quad (6)$$

From (5) and (6), one can see that $\xi_r(i)$ is independent of channel index i, while $\xi_s(i)$ monotonically decreases with i and is independent of the total number of channels $|\mathbf{N}|$. Furthermore, the BSs transmitting in channel n_i follows a PPP of density $\xi_r(i)\lambda_b$ (resp. $\xi_s(i)\lambda_b$) when operating under the random channel assignment scheme (resp. sequential channel assignment scheme).

Finally, we present here some results from [20] which will be widely used in interference calculation of PPP. Given Rayleigh fading with mean $\frac{1}{\mu}$ and the power-law path loss with exponent $\alpha > 2$, the Laplace transform of the total interference power measured at the origin o from a PPP Φ existing outside $\mathbf{B}_o(r)$ and with density λ, can be given by

$$\mathcal{L}_I(s,r,\lambda,p,\mu) = \exp\Big\{-\pi\lambda\big(\frac{\mu}{sp}\big)^{-\frac{2}{\alpha}}\int_{r^2(\frac{\mu}{sp})^{\frac{2}{\alpha}}}^{\infty}\frac{1}{t^{\frac{\alpha}{2}}+1}\mathrm{d}t\Big\} \tag{7}$$

where p is the transmit power adopted by each interferer, and $\mathbf{B}_o(r)$ is the disk centered at the origin o and of radius r.

Remark 2: For the special case of $r=0$, we have

$$\mathcal{L}_I(s,0,\lambda,p,\mu) = \exp\Big\{-\frac{2\pi^2\lambda}{\alpha\sin\frac{2\pi}{\alpha}}\Big(\frac{sp}{\mu}\Big)^{\frac{2}{\alpha}}\Big\}. \tag{8}$$

For the special case of $\alpha=4$, we have

$$\begin{aligned}&\mathcal{L}_I(s,r,\lambda,p,\mu)\\ &= \exp\bigg\{-\pi\lambda\big(\frac{\mu}{sp}\big)^{-\frac{1}{2}}\bigg(\frac{\pi}{2}-\arctan\Big(r^2\big(\frac{\mu}{sp}\big)^{\frac{1}{2}}\Big)\bigg)\bigg\}.\end{aligned} \tag{9}$$

For the special case of $r=0$ and $\alpha=4$, we have

$$\mathcal{L}_I(s,0,\lambda,p,\mu) = \exp\Big\{-\frac{\pi^2\lambda}{2}\Big(\frac{\mu}{sp}\Big)^{-\frac{1}{2}}\Big\}.$$

B. Coverage Probability

For the tagged mobile UE, say u, it has one-hop cellular connection with probability τ_1, and two-hop connection with a BS with probability τ_2. Without loss of generality, we denote by b^* the BS which is closest to u, and denote by u^* the UE closest to u. Then, the coverage probability P_c can be defined as

$$P_c = \tau_1 P_c^1 + \tau_2 P_c^2. \tag{10}$$

(1) Random Channel Assignment

Denoting by η the corresponding SINR threshold required for coverage, we can calculate the coverage probability of one-hop cellular link under the random channel assignment as

$$\begin{aligned}P^1_{c\_r} = &\Big(\sum_{k=1}^{|\mathbf{N}|}\frac{g(k)}{|\mathbf{N}|}+\sum_{k=|\mathbf{N}|+1}^{\infty}\frac{g(k)}{k}\Big)\cdot\\ &\sum_{i=1}^{|\mathbf{N}|}\Pr\big(\mathrm{SINR}_i(b^*,u)>\eta\big).\end{aligned} \tag{11}$$

In (11), $\mathrm{SINR}_i(b^*,u)$ is the SINR received at u from b^* in channel n_i, and $\sum_{k=1}^{|\mathbf{N}|}\frac{g(k)}{|\mathbf{N}|}+\sum_{k=|\mathbf{N}|+1}^{\infty}\frac{g(k)}{k}$ is the probability of u being assigned channel n_i by b^*, which is closely related to the total number of one-hop connected UEs within the PV cell of u.

Since the data transmissions in D2D links use reserved frequency band, there is no mutual interference between cellular links and D2D links. Therefore, when u is assigned channel n_i for downlink data transmission from b^*, the interferers are the surrounding BSs which are transmitting in channel n_i. From (5), we can see that the interfering BSs follow a PPP of density $\xi_r(i)\lambda_b$ under random assignment. We first derive the CCDF of $\mathrm{SINR}_i(b^*,u)$.

$$\begin{aligned}&\Pr\big(\mathrm{SINR}_i(b^*,u)>T\big)\\ &=\mathbf{E}_{r_b}\Big\{\Pr\Big(\frac{p_b h_b r_b^{-\alpha}}{I_c(i)+\sigma^2}>T\big|r_b\Big)\Big\}\\ &=\int_0^{\infty}\Pr\Big(h_b>\frac{Tx^{\alpha}}{p_b}(I_c(i)+\sigma^2)\Big|x\Big)2\pi\lambda_b x e^{-\lambda_b\pi x^2}\mathrm{d}x\\ &=\int_0^{\infty}e^{-\lambda_b\pi x^2}e^{-\frac{\mu_b T x^{\alpha}\sigma^2}{p_b}}\mathbf{E}_{I_c(i)}\{e^{-\frac{\mu_b T x^{\alpha}}{p_b}I_c(i)}\}2\pi\lambda_b x\mathrm{d}x\end{aligned} \tag{12}$$

where $I_c(i)$ is accumulated interference power at u. As $\mathbf{E}_{I_c(i)}\{e^{-\frac{\mu_b T x^{\alpha}}{p_b}I_c(i)}\}$ in (12) equals the Laplace transform of $I_c(i)$ evaluated at $s=\frac{\mu_b T x^{\alpha}}{p_b}$, we have

$$\mathbf{E}_{I_c(i)}\{e^{-\frac{\mu_b T x^{\alpha}}{p_b}I_c(i)}\} = \mathcal{L}_I\Big(\frac{\mu_b T x^{\alpha}}{p_b},x,\xi_r(i)\lambda_b,p_b,\mu_b\Big) \tag{13}$$

Combining (8), (13), and (12), we have

$$\begin{aligned}&\Pr\big(\mathrm{SINR}_i(b^*,u)>T\big)\\ &=\int_0^{\infty}e^{-\Big(\lambda_b\pi x+\frac{\mu_b T\sigma^2 x^{\frac{\alpha}{2}}}{p_b}+\pi\lambda_b\xi_r(i)T^{\frac{2}{\alpha}}x\int_{T^{-\frac{2}{\alpha}}}^{\infty}\frac{\mathrm{d}t}{t^{\frac{\alpha}{2}}+1}\Big)}\pi\lambda_b\mathrm{d}x\end{aligned} \tag{14}$$

Substituting (14) into (11), we obtain the coverage probability of one-hop cellular link under the random channel assignment scheme. Note that as $\xi_r(i)$ is actually independent of i, then $\Pr(\mathrm{SINR}_i(b^*,u)>T)$ is independent of i. Therefore, the $\sum_{i=1}^{|\mathbf{N}|}\Pr(\mathrm{SINR}_i(b^*,u)>\eta)$ in (11) can be further simplified as $|\mathbf{N}|\cdot\Pr(\mathrm{SINR}_i(b^*,u)>\eta)$.

If we denote by b the BS to which u^* connects, then we can determine the coverage probability of two-hop connection as

$$\begin{aligned}P^2_{c\_r} = &\Big(\sum_{k=1}^{|\mathbf{N}|}\frac{g(k)}{|\mathbf{N}|}+\sum_{k=|\mathbf{N}|+1}^{\infty}\frac{g(k)}{k}\Big)\cdot\\ &\sum_{i=1}^{|N|}\Pr\Big(\mathrm{SINR}_i(b,u^*)>\eta,\mathrm{SINR}(u^*,u)>\eta\Big)\\ &=\Big(\sum_{k=1}^{|\mathbf{N}|}\frac{g(k)}{|\mathbf{N}|}+\sum_{k=|\mathbf{N}|+1}^{\infty}\frac{g(k)}{k}\Big)\cdot\\ &\Big(\sum_{i=1}^{|N|}\Pr\big(\mathrm{SINR}_i(b,u^*)>\eta\big)\Big)\cdot\Pr\big(\mathrm{SINR}(u^*,u)>\eta\big)\end{aligned} \tag{15}$$

where (15) follows after the fact that $\mathrm{SINR}_i(b,u^*)$ and $\mathrm{SINR}(u^*,u)$ are mutually independent.

It is noticed that the $\Pr(\mathrm{SINR}_i(b,u^*)>T)$ in (15) is actually the same as $\Pr(\mathrm{SINR}_i(b^*,u)>T)$ derived in (14). Following a similar derivation, we can derive the CCDF of

$\mathrm{SINR}(u^*, u)$ as

$$\Pr\left(\mathrm{SINR}(u^*, u) > T\right) = \int_0^\infty e^{-\left(\pi\tau_1\lambda_u x + \frac{\mu_u T\sigma^2 x^{\frac{\alpha}{2}}}{p_u} + \frac{\pi\tau_2\lambda_u T^{\frac{2}{\alpha}} x}{|\mathbf{M}|}\int_{T^{-\frac{2}{\alpha}}}^\infty \frac{\mathrm{d}t}{t^{\frac{\alpha}{2}}+1}\right)} \pi\tau_1\lambda_u \mathrm{d}x \tag{16}$$

Therefore, based on (16), (14), (15), (11), and (10), we obtain the coverage probability of a generic mobile UE under the random channel assignment scheme.

(2) Sequential Channel Assignment

Note that when operating under the sequential channel assignment, the probability of UE u being assigned channel n_i by BS b^* is $\sum_{k=i}^\infty \frac{g(k)}{k}$. Following a derivation similar to that for random channel assignment, we have the coverage probability of one-hop cellular link

$$P_{c\_s}^1 = \sum_{i=1}^{|\mathbf{N}|}\Big(\sum_{k=i}^\infty \frac{g(k)}{k}\Big)\Pr\left(\mathrm{SINR}_i(b^*, u) > \eta\right) \tag{17}$$

and the coverage probability of two-hop connection

$$P_{c\_s}^2 = \bigg(\sum_{i=1}^{|N|}\Big(\sum_{k=i}^\infty \frac{g(k)}{k}\Big)\Pr(\mathrm{SINR}_i(b, u^*) > \eta)\bigg)\cdot \Pr(\mathrm{SINR}(u^*, u) > \eta) \tag{18}$$

where $\Pr(\mathrm{SINR}(u^*, u) > \eta)$ follows after (16) by replacing T with η, and $\Pr(\mathrm{SINR}_i(b, u^*) > \eta)$ follows after (14) by replacing T with η and replacing $\xi_r(i)\lambda_b$ with $\xi_s(i)\lambda_b$. Note that under the sequential channel assignment, $\xi_s(i)$ monotonically decreases with the channel index i while being independent of the number of total channels $|\mathbf{N}|$. Therefore, the probability of $\Pr(\mathrm{SINR}_i(b, u^*) > \eta)$ monotonically increases with the channel index i.

C. Ergodic Rate

In this section, we compute the average rate in units of nats/s/Hz for a typical mobile UE, which is able to achieve the Shannon bound for its received instantaneous SINR. We denote by R the average rate, by R_c the average rate of cellular link, and by R_d the average rate of D2D link, then we have

$$R = \tau_1 R_c + \tau_2 \min(R_c, R_d). \tag{19}$$

(1) Random Channel Assignment

We first derive R_c as

$$\begin{aligned} R_c &= \theta\int_0^\infty e^{-\lambda_b\pi x^2}\mathbf{E}\big[\ln(1+\mathrm{SINR}_i(b^*, u))\big]2\pi\lambda_b x\mathrm{d}x \\ &= \theta\int_0^\infty e^{-\lambda_b\pi x^2}\int_0^\infty \Pr(\mathrm{SINR}_i(b^*, u) > e^t - 1)\mathrm{d}t 2\pi\lambda_b x\mathrm{d}x \\ &= \theta\int_0^\infty e^{-\lambda_b\pi x^2} q_{c\_r}(x) 2\pi\lambda_b x\mathrm{d}x \end{aligned} \tag{20}$$

where

$$\theta = \frac{|\mathbf{N}|}{|\mathbf{M}|+|\mathbf{N}|}\frac{\sum_{k=1}^{|\mathbf{N}|}\frac{g(k)}{|\mathbf{N}|} + \sum_{k=|\mathbf{N}|+1}^\infty \frac{g(k)}{k}}{1-g(0)} \tag{21}$$

and

$$q_{c\_r}(x) = \int_0^\infty e^{-\frac{\mu_b x^\alpha \sigma^2(e^t-1)}{p_b}}\mathbf{E}_{I_c(i)}\{e^{-\frac{\mu_b x^\alpha(e^t-1)}{p_b}I_c(i)}\}\mathrm{d}t. \tag{22}$$

In (22), $\mathbf{E}_{I_c(i)}\{e^{-\frac{\mu_b x^\alpha(e^t-1)}{p_b}I_c(i)}\}$ follows after (13) by substituting $T = e^t - 1$.

Similarly, we have

$$R_d = \frac{1}{|\mathbf{M}|+|\mathbf{N}|}\int_0^\infty e^{-\tau_1\lambda_u\pi x^2} q_d(x) 2\pi\tau_1\lambda_u x\mathrm{d}x \tag{23}$$

where

$$q_d(x) = \int_0^\infty e^{-\frac{\mu_u x^\alpha \sigma^2(e^t-1)}{p_u}}\mathbf{E}_{I_d(i)}\{e^{-\frac{\mu_u x^\alpha(e^t-1)}{p_u}I_d(i)}\}\mathrm{d}t \tag{24}$$

and

$$\mathbf{E}_{I_d(i)}\{e^{-\frac{\mu_u x^\alpha(e^t-1)}{p_u}I_d(i)}\} = \mathcal{L}_I\Big(\frac{\mu_u x^\alpha(e^t-1)}{p_u}, x, \frac{\tau_2\lambda_u}{|\mathbf{M}|}, p_u, \mu_u\Big). \tag{25}$$

Combining (25), (23), (20), and (19), we obtain the mean rate R.

(2) Sequential Channel Assignment

The derivation of R under the sequential channel assignment is similar to that for the random channel assignment, except for the derivation of R_c. Note that under sequential channel assignment, the SINR in channel i is closely related to $\xi_s(i)$ and thus the channel index i. Therefore, we can derive R_c as

$$\begin{aligned} R_c &= \frac{1}{|\mathbf{M}|+|\mathbf{N}|}\sum_{i=1}^{|\mathbf{N}|}\frac{\sum_{k=i}^\infty \frac{g(k)}{k}}{1-g(0)}\cdot \int_0^\infty e^{-\lambda_b\pi x^2}\mathbf{E}\big[\ln(1+\mathrm{SINR}_i(b^*, u))\big]2\pi\lambda_b x\mathrm{d}x \\ &= \frac{1}{|\mathbf{M}|+|\mathbf{N}|}\sum_{i=1}^{|\mathbf{N}|}\frac{\sum_{k=i}^\infty \frac{g(k)}{k}}{1-g(0)}\int_0^\infty e^{-\lambda_b\pi x^2} q_{c\_s}(x) 2\pi\lambda_b x\mathrm{d}x \end{aligned} \tag{26}$$

where

$$q_{c\_s}(x) = \int_0^\infty e^{-\frac{\mu_b x^\alpha \sigma^2(e^t-1)}{p_b}}\mathbf{E}_{I_c(i)}\{e^{-\frac{\mu_b x^\alpha(e^t-1)}{p_b}I_c(i)}\}\mathrm{d}t. \tag{27}$$

In (27), $\mathbf{E}_{I_c(i)}\{e^{-\frac{\mu_b x^\alpha(e^t-1)}{p_b}I_c(i)}\}$ follows after (13) by substituting $T = e^t - 1$ and replacing $\xi_r(i)$ with $\xi_s(i)$.

The derivation of R_d under the sequential channel assignment is the same as that in (23).

IV. Numerical Results and Discussions

A. Parameter Settings

In the following numerical results, network parameters were selected according to the LTE instruction [23]. Unless otherwise specified, we consider an interference-limited network with carrier frequency 2 GHz and set the path loss exponent $\alpha = 4$ to represent the typical urban macrocell environment. According to the key parameters for 3GPP Case 1 and Case 3 models in Table 26.3 [23], we set the transmit power of

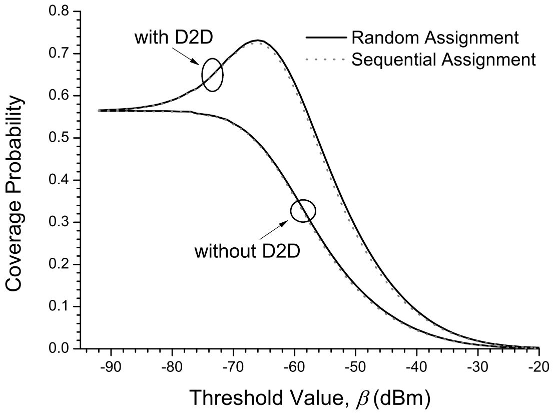


Fig. 3. Impact of the threshold value β on the coverage probability of network with and without D2D communications.

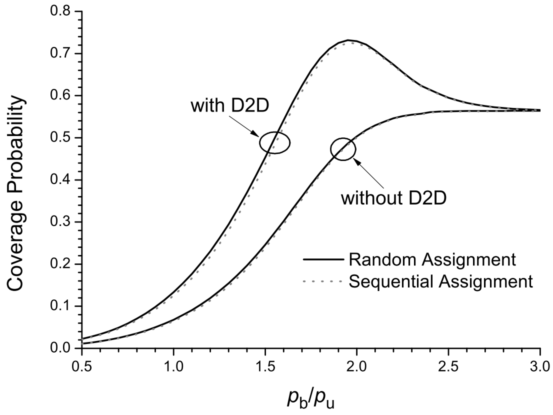


Fig. 4. Impact of $\frac{p_b}{p_u}$ on the coverage probability of network with and without D2D communications, $p_u = 24\,\text{dBm}$.

BSs as $p_b = 46\,\text{dBm}$ (i.e., 39.8 W, a typical value for bands 10, 15, 20 MHz given in the specification 3GPP TS 36.942 and among most manufacturers), and set the transmit power of UEs as $p_u = 24\,\text{dBm}$ (i.e., 251 mW, the maximum output from a UMTS/3G mobile phone). The spatial density of BSs was chosen as $\lambda_b = 1\,\text{BS/km}^2$ which results in an average inter-site distance (ISD) of about 1074 m, and the spatial density of mobile UEs was chosen as $\lambda_u = 50$ UE/km^2. We set the number of channels as $|\mathbf{N}| = 30$, $|\mathbf{M}| = 5$, and assume the fading coefficients for all cellular links and D2D links as $\mu_b = 1$ and $\mu_u = 1$, respectively. The threshold for selecting one-hop connection is $\beta = -65$ dBm, and the SIR threshold for coverage was $\eta = -10\,\text{dB}$. The numerical results under other parameter settings can also be obtained by our analytical framework as well.

B. Coverage Probability

Fig. 3 shows how the coverage probability of network with and without D2D communications, i.e., P_c and $\tau_1 P_c^1$, respectively, varies with the threshold value β. One can easily observe from Fig. 3 that the coverage probability of random channel assignment and that of sequential channel assignment have very similar varying tendencies with β, and the former is slightly bigger than the latter. For the case of without D2D communications, one can see that the coverage probability of both channel assignment schemes monotonically decreases with β. While for the case of with D2D communications, there exists an optimal setting of $\beta = -66\,\text{dBm}$ at which a maximum coverage probability of around 0.73 is achieved. The optimality of β can be interpreted as follows: when β is too small, almost all mobile UEs connects to BSs directly, and thus the number of D2D communications is rather rare; when β is too big, only few mobile UEs that are very close to BSs have direct cellular connection, resulting in rare D2D communications either.

Fig. 4 illustrates how the coverage probability varies with the ratio of transmit power $\frac{p_b}{p_u}$. Interestingly, one can observe from Fig. 4 that for the case of with D2D communications, there also exists an optimal setting of $\frac{p_b}{p_u} = 1.95$ resulting in a maximum coverage probability of around 0.72. Given $p_u = 24\,\text{dBm}$, we obtain the optimal transmit power of BS $p_b = 46.8\,\text{dBm}$, which is actually very close to that given by the specification 3GPP TS 36.942 and among most manufacturers ($46\,\text{dBm}$, i.e., $39.8\,\text{W}$), validating the optimality of the transmit power specified by 3GPP and UMTS. Actually, the effects of increasing $\frac{p_b}{p_u}$ are two folds: on one side, it improves the SINR value at a one-hop connected mobile UE (as indicated by the coverage probability of without D2D communications, which monotonically increases with $\frac{p_b}{p_u}$); on the other side, it has non-negligible effects on the number of established D2D links, just similar to the impact of decreasing β on the number of D2D links (as illustrated in Fig. 2). A further careful observation of Fig. 4 indicates that for both random assignment and sequential assignment, the curve of with D2D communications and that of without D2D communications converge together as $\frac{p_b}{p_u}$ increases beyond 3.0. This is because that when the transmit power of BS p_b increases beyond a threshold, all mobile UEs will connect to BSs directly, and there will be no D2D communications.

Fig. 5 shows the relationship between the coverage probability and the ratio of UE density λ_u to BS density λ_b, given $\lambda_b = 1\,/\text{km}^2$. It is very interesting to observe from Fig. 5 that for both cases of with and without D2D communications, there exist optimal settings of $\frac{\lambda_u}{\lambda_b}$ to achieve the maximum coverage probability. Specifically, for the case of without D2D communication, a maximum coverage probability of 0.56 (resp. 0.536) is achieved at around $\frac{\lambda_u}{\lambda_b} = 15$ (resp. $\frac{\lambda_u}{\lambda_b} = 19$) under random channel assignment (resp. sequential channel assignment); for the case of with D2D communication, a maximum coverage probability of 0.86 (resp. 0.823) is achieved at around $\frac{\lambda_u}{\lambda_b} = 15$ (resp. $\frac{\lambda_u}{\lambda_b} = 19$) under random channel assignment (resp. sequential channel assignment). Therefore, from the perspective of network designers, it is necessary to determine the optimal BS density (i.e., the optimum number

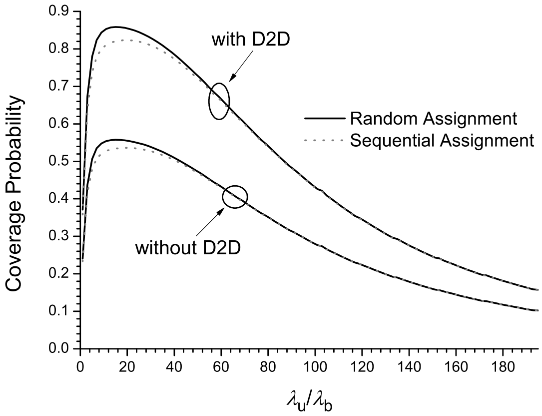


Fig. 5. Impact of $\frac{\lambda_u}{\lambda_b}$ on the coverage probability of network with and without D2D communications, $\lambda_b = 1\,/\text{km}^2$.

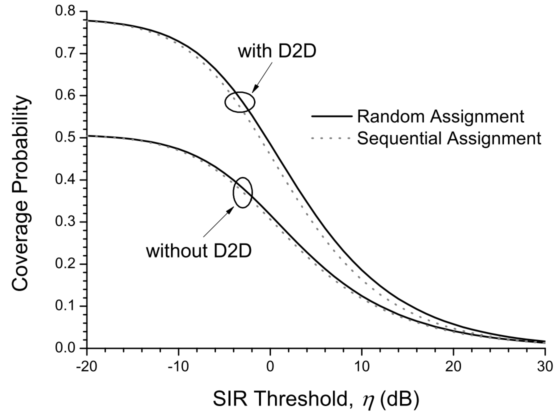


Fig. 7. Impact of SIR threshold η on the coverage probability of network with and without D2D communications.

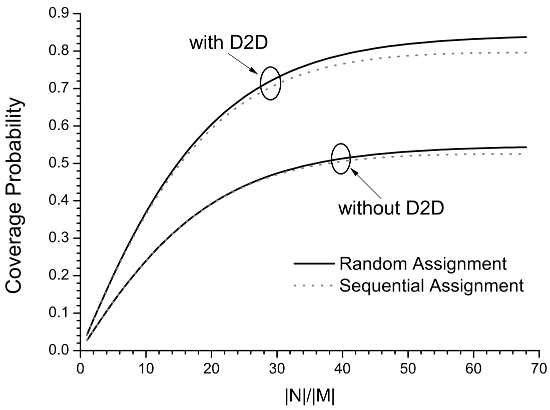


Fig. 6. Impact of $\frac{|\mathbf{N}|}{|\mathbf{M}|}$ on the coverage probability of network with and without D2D communications, $|\mathbf{M}| = 1$.

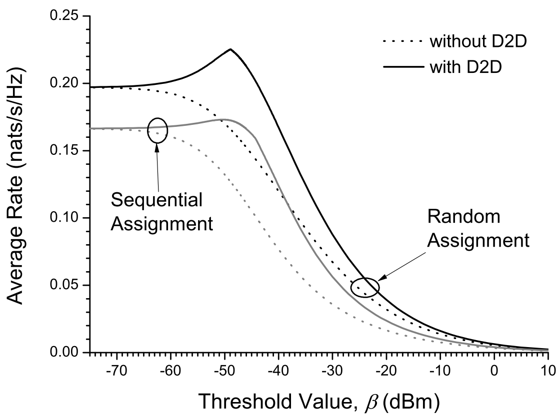


Fig. 8. Impact of threshold β on the average rate of network with and without D2D communications.

of BSs deployed in a unit area), given the approximate number of mobile UEs in a cellular network.

Fig. 6 illustrates how the coverage probability varies with the ratio of channels $\frac{|\mathbf{N}|}{|\mathbf{M}|}$ when $|\mathbf{M}| = 1$, i.e., only one channel is reserved for D2D communication. One can easily observe from Fig. 6 that for both cases of with and without D2D communications, the coverage probability monotonically increases with $\frac{|\mathbf{N}|}{|\mathbf{M}|}$, i.e., more channel resources improve the coverage probability. A further careful observation of Fig. 6 indicates that as $\frac{|\mathbf{N}|}{|\mathbf{M}|}$ increases, the increasing tendency for both curves of with and without D2D communications decreases gradually. The 'saturated' behavior of sequential channel assignment is much more obvious than that of random channel assignment, the coverage probability of which converges to 0.525 (resp. 0.795) for the case of without D2D communication (resp. with D2D communication), as $\frac{|\mathbf{N}|}{|\mathbf{M}|}$ increases beyond 60. Such saturated behavior of coverage probability is attributed to the fixed spatial density of UEs and BSs, as well as the fixed transmit power of UEs and BSs. Another interesting observation of Fig. 6 shows that the gap between random channel assignment and sequential channel assignment increases with $\frac{|\mathbf{N}|}{|\mathbf{M}|}$.

Fig. 7 illustrates the impact of SIR threshold η on the coverage probability. One can observe from Fig. 7 that although both curves of with D2D and without D2D monotonically decrease with η, the coverage probability of the former curve is much bigger than that of the latter curve, which further validates that employing D2D is able to improve the coverage performance. A further careful observation of Fig. 7 indicates that the performance gains of D2D communication in terms of coverage probability, gradually diminish as η increases. Therefore, employing D2D communications is much more effective on improving coverage in a cellular network for the low SIR regime.

C. Ergodic Rate

Figs. 8 and 9 illustrate how the average rates vary with the control parameters β and $\frac{p_b}{p_u}$, respectively. One can easily

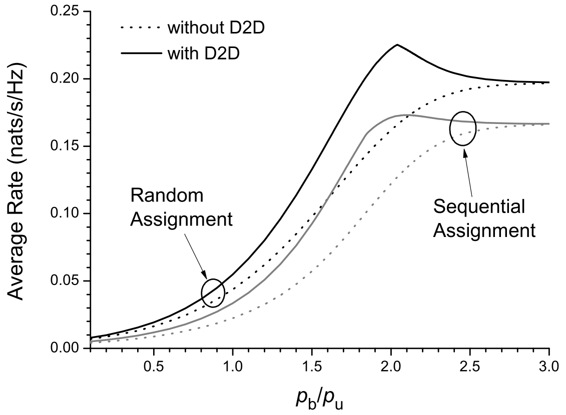


Fig. 9. Impact of $\frac{p_b}{p_u}$ on the average rate of network with and without D2D communications.

observe from Figs. 8 and 9 that, different from the results presented in Section IV-B, the performance gap between random assignment and sequential assignment is non-negligible. In particular, for both the cases of with and without D2D communications in Fig. 8 (resp. in Fig. 9), random assignment is able to achieve a much higher average rate than sequential assignment, especially for the regime of small β (resp. for the regime of high $\frac{p_b}{p_u}$).

A further careful observation of Figs. 8 and 9 indicates that, there exist optimum settings of β and $\frac{p_b}{p_u}$ to maximize the per user average rate of network with D2D communications; while for the case of without D2D communications, there exists no such optimum parameter setting. Furthermore, the performance gain achieved by D2D communication in Figs. 8 and 9, can also be maximized by properly tuning the parameters β and $\frac{p_b}{p_u}$. Actually, it can be interpreted as follows: when β (or $\frac{p_u}{p_b}$) is too small (resp. too big), all nodes (resp. no nodes) choose to establish one-hop connection and thus no two-hop connection can be established.

V. Conclusions

This paper provided a stochastic geometry based theoretical framework to analyze the coverage probability and ergodic rate for a D2D overlaying multi-channel downlink cellular network, which enables network designers to optimize network performances by efficiently determining the optimal network parameters, such as the threshold for mode selection (β), the BS density (λ_b), as well as the transmit power of BSs (p_b). Our results showed that the network coverage can be significantly improved by enabling the combined two-hop connection via an intermediate UE. As future work, we will explore the D2D underlaying multi-tier multi-channel downlink cellular networks, which is much more complicated due to the mutual interference between D2D links and multi-tier cellular links.

Acknowledgment

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