Capacity and Delay of Probing-Based Two-Hop Relay in MANETs

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***Abstract*—Due to their simplicity and efficiency, the two-hop relay algorithm and its variants serve as a class of attractive routing schemes for mobile ad hoc networks (MANETs). With the available two-hop relay schemes, a node, whenever getting an opportunity for transmission, randomly probes only once a neighbor node for the possible transmission. It is notable that such single probing strategy, although simple, may result in a significant waste of the precious transmission opportunities in highly dynamic MANETs. To alleviate such limitation for a more efficient utilization of limited wireless bandwidth, this paper proposes a more general probing-based two-hop relay algorithm with limited packet redundancy. In such an algorithm with probing round limit τ and packet redundancy limit f, each transmitter is allowed to conduct up to τ rounds of probing for identifying a possible receiver and each packet can be delivered to at most f distinct relays. A general theoretical framework is further developed to help us understand that under different setting of τ and f, how we can benefit from multiple probings in terms of the per node throughput capacity and the expected end-to-end packet delay.**

***Index Terms*—Mobile ad hoc networks, two-hop relay, probing, packet redundancy, throughput capacity, end-to-end delay.**

I. INTRODUCTION

As nodes move around randomly in mobile ad hoc networks (MANETs), network topology varies dramatically and there may exist no contemporaneous end-to-end path at any given time instant [1]–[5]. As a consequence, the traditional route-based routing schemes like DSR [6], AODV [7], etc., fail to function properly since they require the simultaneous availability of a number of links. The two-hop relay routing which takes advantage of node mobility and sequences of node contacts to deliver messages from end to end, since first introduced in [8], has become a promising routing protocol for MANETs [1], [9], [10]. As shown in [1], [10], [11], the two-hop relay and its variants, simple yet efficient, are able to provide a flexible control of both the throughput and packet delay for the challenging MANETs. Under such a routing scheme, a packet reaches its destination either through a direct transmission from the source or by two-hop transmissions via an intermediate relay node, which first receives the packet from the source and then forwards it to the destination. Therefore, each packet travels at most two hops to reach the destination.

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The two-hop relay algorithm and its variants have been intensively studied in literature. The algorithms in [8], [12]–[17] can be regarded as the out-of-order routing without packet redundancy, where a packet has at most one copy and will be accepted by its destination as long as it has never been received before. The two-hop relays in [1], [18]–[25] also adopt the out-of-order reception but multiple redundant copies can be distributed for each packet. Later, some new two-hop relay algorithms with in-order reception have been proposed in [26]–[28] where each packet has a fixed number of copies (i.e., with exact redundancy). The two-hop relay schemes in [9], [11], [29] also belongs to the line of in-order reception but each packet is allowed to have a limited number of copies (i.e., with limited redundancy). More recently, a general group-based two-hop relay with limited redundancy was also proposed in [30], where each packet is delivered to a limited number of relay nodes and can be accepted by its destination if it is among the group of packets the destination is currently requesting.

Notice that in the available two-hop relay schemes with packet redundancy (fixed or limited), no matter adopting out-of-order reception [1], [18]–[25], in-order reception [9], [11], [26]–[29] or group-based reception [30], a node, whenever getting an opportunity for transmission, randomly probes only once a neighbor node for possible transmission if its destination node is not within its transmission range. Such single probing strategy, although simple, may result in a significant waste of the precious transmission opportunities in highly dynamic MANETs. For example, for the case that the transmitter regards a randomly probed neighbor node as a relay and hopes to deliver a redundant packet copy to it, it may happen that the relay is already carrying such a copy for that packet; on the other hand, for the case that the transmitter acts as a relay and hopes to forward a packet to the randomly probed node, this node may have already received all the packets carried by the transmitter. Thus, when a wrong node is selected through such single probing strategy, no transmission can be conducted successfully in the above two cases and the transmission opportunity of the transmitter will be wasted.

To alleviate the limitation of single probing for a more efficient utilization of wireless bandwidth, this paper considers a general probing-based two-hop relay with limited packet redundancy. The main contributions of this paper are summarized as follows:

- We propose a new two-hop relay algorithm with probing round limit τ and packet redundancy limit f (2HR-(τ, f) for short), where each transmitter is allowed to conduct up

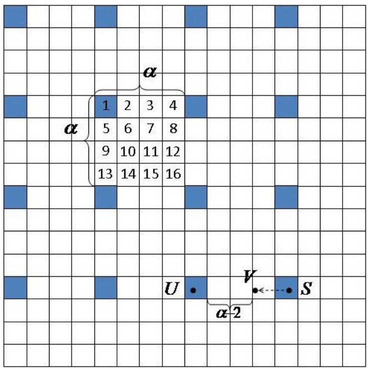

Fig. 1. Illustration of cells in a transmission-group with $m = 16$ and $\alpha = 4$.

to τ rounds of probing for identifying a possible receiver and each packet can be delivered to at most f distinct relays. This algorithm covers available two-hop routing protocols [9], [11], [26], [27] as special cases ($\tau = 1$).

- We further develop a general theoretical framework to characterize the complicated packet delivery process under the 2HR-(τ, f), where the finite-state absorbing Markov chain technique is adopted to model the packet dispatching process at the source and the packet receiving process at the destination. By setting $\tau = 1$, our framework reduces to some available models developed for two-hop relay [9], [11].
- With the help of the theoretical framework, closed-form expressions are derived for the per node throughput capacity and the expected end-to-end packet delay. Extensive simulation and theoretical results are also provided to validate the efficiency of the new relay algorithm and corresponding theoretical framework.

The remainder of this paper is outlined as follows. In Section II, we introduce the system models, the transmission-group based scheduling scheme and the 2HR-(τ, f) routing algorithm. In Section III we develop Markov chain framework and derive closed-form expressions for the per node throughput capacity and the expected end-to-end packet delay for any feasible traffic input rate. We provide numerical results to validate our scheme and theoretical framework in Section IV, introduce the available works regarding throughput capacity and delay analysis in Section V and conclude the whole paper in Section VI.

II. 2HR-(τ, f) Routing Algorithm

A. System models

Similar to [12], [31], we consider in this paper a torus network of unit area which is evenly divided into $m \times m$ cells. Fig. 1 shows an example of a 16×16 cell-partitioned network where the cells are further divided into 16 distinct transmission-groups, and all the shaded cells there belong to the same transmission-group (as to be introduced later in Section II-B). Time is slotted, and there are n nodes roaming around in the torus from cell to cell according to the i.i.d. mobility model [9], [26]. Each node employs a common transmission range r, and the protocol model [32] with guard factor Δ is adopted here to account for interference issues. A whole time slot is allocated only for data transmissions in one-hop range, and for any node pair the data bits that can be successfully transmitted from the transmitter to the receiver is normalized to one packet here. We consider the permutation traffic pattern widely adopted in previous studies [8], [9], [31], where there are in total n distinct traffic flows (one flow corresponds to one source-destination pair). Under such traffic pattern, each node is not only the source of its locally generated traffic flow but also the destination of another traffic flow originated from some other node. The traffic flow generated at each node is assumed to have an average input rate λ (packets/slot).

B. Scheduling Scheme

We consider a local transmission scenario where a transmitter in some cell can only transmit packets to receivers in the same cell or its eight neighboring cells. Two cells are called neighboring cells if they share a common point. Thus, the transmission range can be determined as $r = \sqrt{8}/m$. According to the protocol interference model, two links can transmit simultaneously if and only if they are sufficiently far away from each other. To avoid collisions among simultaneous transmissions and schedule as many simultaneous link transmissions as possible, similar to [9], [33]–[35], we adopt the transmission-group based scheduling scheme.

Transmission-group: A transmission-group is a subset of cells where any two of them have a vertical and horizontal distance of some multiple of α cells and all the cells there could transmit simultaneously without interfering with each other.

It is easy to see that under the transmission-group based scheduling scheme, all the m^2 cells in the network can be divided into α^2 distinct transmission-groups. If we let each transmission-group become active (i.e., have link transmissions) alternatively, then each cell will also become active every α^2 time slots.

As illustrated in Fig. 1 for the case $\alpha = 4$, there are in total 16 transmission groups, and all shaded cells belong to the same transmission-group. Suppose that the transmission-group 1 is active, and node S in some active cell is transmitting to node V. Then the distance between node V and some other transmitter in another active cell is at least $\alpha - 2$ cells. According to the protocol interference model, in order to ensure the successful data reception at node V, we should have $(\alpha - 2) \cdot \frac{1}{m} \geq (1 + \Delta) \cdot r$. Notice that $\alpha \leq m$, then the parameter α can be determined as

$$\alpha = \min\{\lceil (1 + \Delta)\sqrt{8} + 2 \rceil, m\} \qquad (1)$$

Now we proceed to introduce the partition of a time slot. As shown in Fig. 2, each time slot is divided into four subslots. In subslot W_1, all nodes in an active cell contend to become the transmitter in a DCF way, where each node there randomly selects a back-off counter from $(0, W_1]$ and the node whose counter is the first to become zero broadcasts a message claiming itself as the transmitter. Subslot W_2 is specified for destination checking where the destination node of the flow

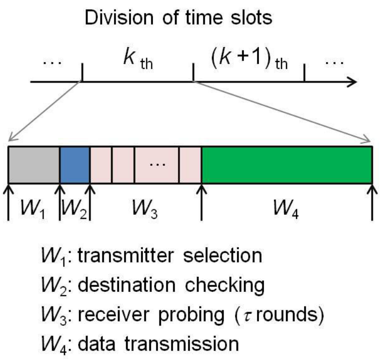

Fig. 2. Partition of a time slot.

originated from the transmitter will reply to the transmitter if it is inside the one-hop neighborhood. Otherwise, if no reply is heard from the destination, in subslot W_3 the transmitter will conduct at most τ rounds of probing until an eligible receiver is selected (in each probing round, a neighboring node is randomly selected as the receiver). Subslot W_4 is reserved for data transmission from the transmitter to the selected receiver. If no eligible receiver is selected in subslot W_3 (and thus no packet can be transmitted), the transmitter stays idle in subslot W_4.

C. 2HR-(τ, f) Routing Algorithm

Now we are ready to introduce the general probing-based two-hop relay algorithm 2HR-(τ, f). Under such an algorithm, each transmitter will conduct at most τ rounds of probing to select an eligible receiver when its destination node is not inside the one-hop neighborhood, and at most f copies will be distributed out for each packet.

Notice that under the permutation traffic pattern considered in this paper, there are in total n distinct flows and each node can be a potential relay for other $n-2$ flows (excluding the two flows originated from and destined for itself). We assume that each node maintains in its buffer n individual FIFO queues: one local-queue storing the locally generated packets, one already-sent-queue storing the packets whose f copies have been distributed but the reception status are not confirmed yet, and $n-2$ relay-queues storing packets from other $n-2$ flows (one for each flow). For throughput capacity analysis, we assume all queues have enough buffer space such that no packet overflow will happen.

Without loss of generality, we focus on a tagged flow and denote by S the source node and denote by D the destination node. We consider a scenario where the source S and the destination D use the push-type of service for data transmission. Specifically, S periodically sends locally generated new packets to D via intermediate relay nodes; D can only passively receive packets from S (or relay nodes) and never sends any request to S.

It is noticed that for the designing of relay algorithm with packet redundancy, one common complication is that remnant copies of a packet that has already been received at its destination create excess congestion and must somehow be removed. Together with the push-type data transmission, another complication is that the transmitter has no idea which packet should be sent to the receiver during each node meeting, since it may happen that the transmitter initially plans to send a packet say P, but the receiver has already received a copy of packet P from another relay node (or the source). In order to overcome these two complications, we adopt a sequence number based mechanism for the 2HR-(τ, f) algorithm. For the tagged flow, the source S labels each packet P waiting at the local-queue with a *sequence number* $SN(P)$ to represent its arrival order, and the destination D maintains an *indicator number* $IN(D)$ to denote that D has received all packets with sequence number less than $IN(D)$. Every time node S (resp. node D) moves ahead its local-queue by one packet (resp. receives a packet), it increases its sequence number (resp. indicator number) by one. Therefore, each packet is received in order at the destination.

Every time S is selected as the transmitter in an active cell, it executes the following Algorithm 1.

Algorithm 1 2HR-(τ, f) routing algorithm

```
S checks whether its destination D is in the one-hop neighborhood;
if D is within the one-hop neighborhood of S then
    S executes Procedure 1;
else
    With probability 1/2, S randomly selects to do source-to-relay transmission or relay-to-destination transmission;
    if S selects source-to-relay transmission then
        S executes Procedure 2;
    else
        S executes Procedure 3;
    end if
end if
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Procedure 1 source-to-destination transmission

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S obtains from D the indicator number IN(D);
S directly sends to D the packet P with sequence number SN(P) = IN(D);
S deletes all packets with sequence number less than IN(D) from both local-queue and already-sent-queue;
S moves ahead the remaining packets in local-queue and already-sent-queue;
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Remark 1: It is noticed that in Procedure 1, after obtaining the indicator number $IN(D)$ from the destination D, the source S deletes all packets with sequence number less than $IN(D)$ and tries to send a packet P with sequence number $SN(P) = IN(D)$ to D. Similarly, in Procedure 3, S conducts similar buffer update and packet transmission after obtaining the $IN(V_i)$ from node V_i. The above handshake process in Procedures 1 and 3 happens only before the data transmission during each node meeting, and a receiver will send its indicator number to the transmitter only during such handshake process.

Procedure 2 source-to-relay transmission

$i \leftarrow 1$;
while $i \leq \tau$ **do**
  S randomly selects a node (say V_i) out of the one-hop neighbors;
  S checks whether the head-of-line (HoL) packet P_h at its local-queue is carried by V_i;
  if V_i doesn't carry P_h **then**
    S delivers to V_i a copy of P_h;
    if All f copies of P_h have been distributed **then**
      S puts P_h into the end of the already-sent-queue;
      S moves ahead the remaining packets behind P_h in the local-queue;
    end if
    $i \leftarrow \tau + 1$;
  end if
  $i \leftarrow i + 1$;
end while

Procedure 3 relay-to-destination transmission

$i \leftarrow 1$;
while $i \leq \tau$ **do**
  S randomly selects a node (say V_i) out of the one-hop neighbors;
  S obtains from V_i the indicator number $IN(V_i)$;
  S checks whether it carries a packet P with $SN(P) = IN(V_i)$ in its relay-queue specified for V_i;
  if S carries such packet P **then**
    S sends P to node V_i;
    S deletes all packets with sequence number less than $IN(V_i)$ from its relay-queue specified for V_i;
    S moves ahead the remaining packets in the relay-queue specified for V_i;
    $i \leftarrow \tau + 1$;
  end if
  $i \leftarrow i + 1$;
end while

III. Throughput Capacity And Expected End-to-end Delay

In this section, we first introduce some basic probabilities and explore the service times at the source S and the destination D, then proceed to derive the throughput capacity and the expected end-to-end delay under the general 2HR-(τ, f) algorithm.

A. Some Basic Probabilities

Lemma 1: Consider a MANET adopting the 2HR-(τ, f) routing algorithm. For a given time slot and the tagged flow, if we use p_1 to denote the probability that the source S conducts a source-to-destination transmission and use p_2 to denote the probability that S conducts a source-to-relay or relay-to-destination transmission, then we have

$$p_1 = \frac{1}{\alpha^2}\left\{\frac{9n-m^2}{n(n-1)} - \left(\frac{m^2-1}{m^2}\right)^{n-1}\frac{8n+1-m^2}{n(n-1)}\right\} \tag{2}$$

$$p_2 = \frac{1}{\alpha^2}\left\{\frac{m^2-9}{n-1}\left(1-\left(\frac{m^2-1}{m^2}\right)^{n-1}\right) - \left(\frac{m^2-9}{m^2}\right)^{n-1}\right\} \tag{3}$$

Proof: The derivations of probabilities p_1 and p_2 under the probing-based 2HR-(τ, f) routing algorithm, are similar to that in [9]. Please kindly refer to [9] for details. ∎

According to Procedure 2 of the 2HR-(τ, f) routing algorithm, we can see that when the source node S decides to conduct the source-to-relay transmission, it will independently conduct at most τ rounds of probing (in each probing round, a neighboring node is randomly selected as the receiver) to deliver out a copy for its HoL packet P_h. Then we have the following lemma.

Lemma 2: In a MANET with 2HR-(τ, f) routing algorithm, for a given time slot and the tagged flow, suppose the source S is delivering copies for the HoL packet P_h at its local-queue, and there are already j copies of P_h in the network, $1 \leq j \leq f$. If we denote by $P_d(j)$ the probability that S successfully delivers a new copy of P_h to some relay node in the time slot, then we have

$$P_d(j) = \frac{(m^2-9)^{n-1}}{2\alpha^2 m^{2n-2}}\left\{\sum_{s=1}^{n-j-1}\sum_{t=0}^{j-1}\sum_{k=0}^{s+t}\binom{n-j-1}{s}\binom{j-1}{t}\binom{s+t}{k}\frac{8^{s+t-k}}{(m^2-9)^{s+t}}\cdot\frac{1}{k+1}\cdot\left(1-\left(\frac{t}{t+s}\right)^{\tau}\right)\right\} \tag{4}$$

Now we proceed to explore the probability that the destination node D may receive a packet whose sequence number equals $IN(D)$ in Procedure 3. Consider some relay node R carrying a packet P with $SN(P) = IN(D)$ in its relay-queue specified for D. For a time slot, suppose R is selected as the transmitter and R decides to conduct the relay-to-destination transmission. It is easy to see that R will deliver to D the packet P if and only if the following two events happen simultaneously: D is selected as the receiver in the t_{th} round of probing, $1 \leq t \leq \tau$; for the node V_i selected in the i_{th} round of probing, $1 \leq i < t$, $V_i \neq D$, R does not carry any packet $P^{'}$ with $SN(P^{'}) = IN(V_i)$ in its relay-queue for V_i. Without loss of generality, we denote by p_{nc} the probability that R does not carry any packet $P^{'}$ with $SN(P^{'}) = IN(V_i)$, $1 \leq i < t$, then we have the following lemma.

Lemma 3: In a MANET with 2HR-(τ, f) routing algorithm, for a given time slot and the tagged flow, suppose there are already j copies of packet P with $SN(P) = IN(D)$ in the network, $1 \leq j \leq f+1$. If we denote by $P_r(j)$ the probability that D successfully receives P in the time slot, then we have

$$P_r(j) = p_1 + \frac{(j-1)(m^2-9)^{n-2}}{2\alpha^2(n^2-3n+2)m^{2n-2}} \cdot \sum_{k=0}^{n-3}\binom{n-1}{k+2}\frac{1-(\frac{k}{k+1}p_{nc})^{\tau}}{1-\frac{k}{k+1}p_{nc}} \cdot \frac{9^{k+2}-8^{k+2}}{(m^2-9)^k} \tag{5}$$

The proofs of Lemmas 2 and 3 can be found in Appendix A.

Remark 2: Our 2HR-(τ, f) routing algorithm covers the routing scheme in [9] as special cases. By setting $\tau = 1$, we have $P_d(j) = \frac{n-j-1}{2(n-2)} \cdot p_2$ and $P_r(j) = p_1 + \frac{j-1}{2(n-2)} \cdot p_2$, which reduce to the results derived in [9].

Actually for a tagged flow under the 2HR-(τ, f) routing algorithm, the packet dispatching process at the source S and the packet receiving process at the destination D can be modeled by the finite-state absorbing Markov chain technique. In the following, we take the packet dispatching process at S as an example to justify why it can be modeled by the finite-state absorbing Markov chain technique. The argument for the packet receiving process at D easily follows in a similar way. Without loss of generality, consider a general packet say P at S.

First, we show that the dispatching process of P at S is a finite-state Markov chain. It is easy to see that for the 2HR-(τ, f) scheme, the dispatching process of P consists of a set of states, $\mathbf{I} = \{h_1, h_2, \ldots, h_f, h_{f+1}\}$, where state h_x denotes that there are x copies of P in the network, $1 \leq x \leq f$, and state h_{f+1} denotes that S either has delivered P to D or has distributed P to f distinct relay nodes. The dispatching process starts from state h_1 and it moves successively from one state to another. Furthermore, if we assume that the dispatching process is in state $H(t)$ at time slot t, $H(t) \in \mathbf{I}$, then we have

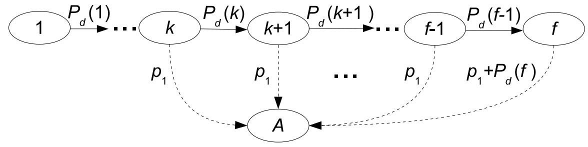
$$\begin{aligned}\Pr\big(H(t+1) = h_x \vert H(1), H(2), \ldots, H(t)\big) \\ = \Pr\big(H(t+1) = h_x \vert H(t)\big), x \in [1, f+1]\end{aligned}$$

Together with the fact that the set $\mathbf{I}$ has a limited number of states ($f+1$ in total), therefore, the dispatching process of P at S can be modeled by a finite-state Markov chain.

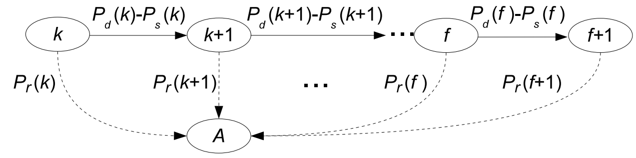
Now we proceed to show that the above Markov chain is actually an absorbing Markov chain. According to the absorbing Markov chain theory [36], [37], we only need to show that the above Markov chain satisfies the following two conditions: 1) there is at least one absorbing state; 2) it is possible to go from any state to at least one absorbing state in a finite number of steps. It is easy to see that the state h_{f+1} is an absorbing state, since once S has delivered P to D or has distributed P to f distinct relay nodes S finishes the copy dispatching process for P. Furthermore, it is possible for the chain to go from any state h_x to the absorbing state h_{f+1} in a single step, $x \in [1, f]$, since S may directly deliver P to D via the source-to-destination transmission with probability p_1 during each time slot (as proved in Lemma 1). Therefore, we prove that when operating under the 2HR-(τ, f) scheme, the packet dispatching process at S satisfies the criteria of mathematical definition of finite-state absorbing Markov chain.

B. Service Times at the Source S and the Destination D

When operating under the 2HR-(τ, f) routing algorithm, for the tagged flow each packet will experience two queuing processes, i.e., the packet dispatching process at the local-queue of the source node S and the packet receiving process at the virtual queue of the destination node D [9], [11]. Since the virtual queue stores the sequence numbers of those packets not received yet by D, the head-of-line entry of the virtual queue always equals the indicator number at D, i.e., $IN(D)$. Before proceeding to derive the service times at S and D, we first formally introduce the following definition.



(a) Absorbing Markov chain for the packet dispatching process at the source node S.



(b) Absorbing Markov chain for the packet receiving process at the destination node D.

Fig. 3. Absorbing Markov chain for a packet P of the tagged flow, given that there are already k copies of P in the network when the entry $SN(P)$ is moved into the head-of-line at the virtual queue. For each transient state, the transition back to itself is not shown for simplicity.

Definition 1: For a general packet P of the tagged flow, its service time at the source S is defined as the time elapsed between the time slot when S starts to deliver copies for P (i.e., when S moves P into the head-of-line at the local-queue) and the time slot when S stops distributing copies for P; the service time of P at the destination D is defined as the time elapsed between the time slot when the indicator number $IN(D)$ is updated to $SN(P)$ (i.e., when the entry $SN(P)$ is moved into the head-of-line at the virtual queue) and the time slot when D receives P.

For a time slot and a general packet P of the tagged flow, suppose that there are already k copies of P (including the original one at the source node S) in the network when the entry $SN(P)$ is moved into the head-of-line at the virtual queue, $1 \leq k \leq f+1$. If we denote by $P_s(k)$ the probability of simultaneous source-to-relay transmission (from S to some node without P) and relay-to-destination transmission (from some relay carrying P to D), we can see that for the packet P, the dispatching process at S and the receiving process at D can be modeled by two finite-state absorbing Markov chains shown in Fig. 3a and 3b, respectively, where the absorbing state A denotes the termination of the service process.

Given there are already k copies of P in the network when the entry $SN(P)$ is moved into the head-of-line at the virtual queue (i.e., D receives the last packet before P), if we denote by $X_S(k)$ the service time at S and denote by $X_D(k)$ the service time at D, it is easy to see that $X_S(k)$ (resp. $X_D(k)$) corresponds to the time it takes the Markov chain in Fig. 3a (resp. in Fig. 3b) to become absorbed given that the chain starts from state 1 (resp. state k).

Lemma 4: In a MANET with 2HR-(τ, f) routing algorithm, for a general packet P of the tagged flow, suppose that there are already k copies of P in the network when the indicator number $IN(D)$ is updated to $SN(P)$, then the expected service times $\mathbb{E}\{X_S(k)\}$ and $\mathbb{E}\{X_D(k)\}$ can be determined as

$$\mathbb{E}\{X_S(k)\} = \begin{cases} \sum_{i=1}^{k-1} \frac{1}{P_d(i)} + \frac{1}{p_1+P_d(k)} \\ \quad \cdot\big(1+\sum_{j=1}^{f-k}\phi_1(k,j)\big) & \text{if } 1 \le k \le f, \\ \sum_{i=1}^{f} \frac{1}{P_d(i)} & \text{if } k = f+1. \end{cases} \tag{6}$$

$$\mathbb{E}\{X_D(k)\} = \begin{cases} \frac{1}{P_r(k)+P_d(k)-P_s(k)}\big(1+\sum_{j=1}^{f-k}\phi_2(k,j) \\ \quad +\frac{P_d(f)-P_s(f)}{P_r(f+1)}\phi_2(k,f-k)\big) \\ \qquad \text{if } 1 \le k \le f-1, \\ \frac{1}{P_r(f)+P_d(f)-P_s(f)}\big(1+\frac{P_d(f)-P_s(f)}{P_r(f+1)}\big) \\ \qquad \text{if } k = f, \\ \frac{1}{P_r(f+1)} \qquad \text{if } k = f+1. \end{cases} \tag{7}$$

where

$$\phi_1(k,j) = \prod_{t=1}^{j} \frac{P_d(k+t-1)}{p_1 + P_d(k+t)}$$

$$\phi_2(k,j) = \prod_{t=1}^{j} \frac{P_d(k+t-1) - P_s(k+t-1)}{P_r(k+t) + P_d(k+t) - P_s(k+t)}$$

Proof: The derivations of expected service times $\mathbb{E}\{X_S(k)\}$ and $\mathbb{E}\{X_D(k)\}$ under the probing-based 2HR-(τ, f) routing algorithm are similar to that in [9], and please refer to [9] for details. ■

Lemma 5: Regarding the expected service times $\mathbb{E}\{X_S(k)\}$ and $\mathbb{E}\{X_D(k)\}$, we have

$$\mathbb{E}\{X_S(k+1)\} > \mathbb{E}\{X_S(k)\}, \quad 1 \le k \le f \tag{8}$$

$$\mathbb{E}\{X_D(k+1)\} < \mathbb{E}\{X_D(k)\}, \quad 1 \le k \le f \tag{9}$$

Proof: As the proof of (8) is similar to that in [9], we omit it here. Before proceeding to prove (9), we first employ the mathematical induction to prove the following inequality

$$P_r(k) \cdot \mathbb{E}\{X_D(k+1)\} < 1, \quad 1 \le k \le f \tag{10}$$

which will be used for the proof of (9).

Initial step: for $k = f$, it is easy to see that

$$P_r(f) \cdot \mathbb{E}\{X_D(f+1)\} = \frac{P_r(f)}{P_r(f+1)} < 1 \tag{11}$$

Inductive step: we assume that (10) holds for some $k = t$, $1 < t \le f$, i.e., $P_r(t) \cdot \mathbb{E}\{X_D(t+1)\} < 1$. We need to prove (10) still holds for $k = t-1$.

$$\begin{aligned} &P_r(t-1) \cdot \mathbb{E}\{X_D(t)\} \\ &= P_r(t-1) \cdot \frac{1 + (P_d(t) - P_s(t)) \cdot \mathbb{E}\{X_D(t+1)\}}{P_r(t) + P_d(t) - P_s(t)} \\ &< \frac{P_r(t-1) + P_r(t)\mathbb{E}\{X_D(t+1)\}(P_d(t) - P_s(t))}{P_r(t) + P_d(t) - P_s(t)} \quad (12) \\ &< \frac{P_r(t-1) + P_d(t) - P_s(t)}{P_r(t) + P_d(t) - P_s(t)} < 1 \quad (13) \end{aligned}$$

where (12) follows after $P_r(t-1) < P_r(t)$. Combining (11) and (13), we prove (10).

Now we are ready to prove (9). For the case that $k = f$, we have

$$\begin{aligned} &\mathbb{E}\{X_D(f)\} - \mathbb{E}\{X_D(f+1)\} \\ &= \frac{1 + \frac{P_d(f)-P_s(f)}{P_r(f+1)}}{P_r(f) + P_d(f) - P_s(f)} - \frac{1}{P_r(f+1)} \\ &= \frac{1}{P_r(f+1)}\left(\frac{P_r(f+1) + P_d(f) - P_s(f)}{P_r(f) + P_d(f) - P_s(f)} - 1\right) > 0 \end{aligned} \tag{14}$$

For the case that $1 \le k < f$, we have

$$\begin{aligned} &\mathbb{E}\{X_D(k)\} - \mathbb{E}\{X_D(k+1)\} \\ &= \frac{1 + (P_d(k) - P_s(k))\mathbb{E}\{X_D(k+1)\}}{P_r(k) + P_d(k) - P_s(k)} - \mathbb{E}\{X_D(k+1)\} \\ &= \frac{1 - P_r(k)\mathbb{E}\{X_D(k+1)\}}{P_r(k) + P_d(k) - P_s(k)} > 0 \end{aligned} \tag{15}$$

where (15) follows after substituting (10). After combining (14) and (15), we have (9) and then complete the proof for Lemma 5. ■

Lemma 6: For the tagged flow, if we denote by $\overline{X}_S$ the mean service time averaged over all packets locally generated at the source S and denote by $\overline{X}_D$ the mean service time averaged over all packets received at the destination D, then we have

$$\mathbb{E}\{X_S(1)\} \le \overline{X}_S \le \mathbb{E}\{X_S(f+1)\} \tag{16}$$

$$\mathbb{E}\{X_D(f+1)\} \le \overline{X}_D \le \mathbb{E}\{X_D(1)\} \tag{17}$$

Proof: From the definitions of $\overline{X}_S$ and $\overline{X}_D$, it is easy to see that (16) and (17) follow directly after (8) and (9), respectively. ■

C. Throughput Capacity of 2HR-(τ, f)

Before deriving the throughput capacity of the proposed 2HR-(τ, f) routing algorithm, we first introduce the following lemma.

Lemma 7: For the 2HR-(τ, f) routing algorithm, $1 \le f \le n-2$, $1 \le \tau \le \tau_0$, we have

$$\mathbb{E}\{X_S(f+1)\} \le \mathbb{E}\{X_D(f+1)\} \tag{18}$$

where τ_0 is given by

$$\tau_0 = \lfloor \frac{(n-f-1)p_2 - 2(n-2)p_1 \cdot f}{p_2 \cdot f^2} \rfloor \tag{19}$$

Proof: It is easy to observe from (4) that

$$P_d(j) \ge \frac{n-j-1}{2(n-2)} \cdot p_2 \tag{20}$$

From (6), we can see that

$$\begin{aligned} \mathbb{E}\{X_S(f+1)\} &= \sum_{j=1}^{f} \frac{1}{P_d(j)} \le \sum_{j=1}^{f} \frac{1}{\frac{n-j-1}{2(n-2)} \cdot p_2} \quad (21) \\ &= \frac{2(n-2)}{p_2} \sum_{j=1}^{f} \frac{1}{n-j-1} \\ &< \frac{2(n-2)}{p_2} \cdot \frac{f}{n-f-1} \quad (22) \end{aligned}$$

where (21) follows after substituting (20).

Similarly from (5), we have

$$P_r(j) \leq p_1 + \tau \cdot \frac{j-1}{2(n-2)} \cdot p_2 \tag{23}$$

Combining (23) and (7), then we have

$$\mathbb{E}\{X_D(f+1)\} = \frac{1}{P_r(f+1)} \geq \frac{1}{p_1 + f \cdot \tau \cdot \frac{p_2}{2(n-2)}} \tag{24}$$

From (24) and (22), we can see that (18) holds if

$$\frac{1}{p_1 + f \cdot \tau \cdot \frac{p_2}{2(n-2)}} \geq \frac{2(n-2)}{p_2} \cdot \frac{f}{n-f-1} \tag{25}$$

after some basic algebraic operations, (19) follows directly after (25). Then we finish the proof for Lemma 7. ■

Theorem 1: In a cell partitioned MANET where nodes move according to the i.i.d. mobility model and the 2HR-(τ, f) is adopted for packet routing, if we denote by μ the per-node throughput capacity, i.e., the network can stably support any traffic input rate $\lambda(\lambda < \mu)$, then for any given f and τ, $1 \leq f \leq n-2$, $1 \leq \tau \leq \tau_0$, the per node throughput capacity μ can be determined as

$$\mu = p_1 + \frac{f \cdot (m^2-9)^{n-2}}{2\alpha^2(n^2-3n+2)m^{2n-2}} \sum_{k=0}^{n-3} \binom{n-1}{k+2} \cdot \frac{9^{k+2}-8^{k+2}}{(m^2-9)^k} \cdot \frac{(k+1)^\tau (n-2)^\tau - k^\tau (n-2-f)^\tau}{(n-2)^{\tau-1}(k+1)^{\tau-1}(n-2+kf)} \tag{26}$$

Proof: From Lemmas 5, 6, 7 and Theorem 1 in [9], we can see that for any given f and τ, $1 \leq f \leq n-2$, $1 \leq \tau \leq \tau_0$, the per node throughput capacity μ is determined as

$$\mu = \frac{1}{\mathbb{E}\{X_D(f+1)\}} = P_r(f+1) \tag{27}$$

combining with (5), we can see that in order to derive the throughput capacity μ, the only remaining issue is to derive the probability p_{nc}.

Notice that according to Theorem 1 in [11], for a general packet P of the tagged flow, as the traffic input rate approaches the throughput capacity, i.e., $\lambda \to \mu$, the destination D receives the last packet before P (i.e., the indicator number $IN(D)$ is updated to $SN(P)$) only after the source node S has already distributed out all f copies for P. If we denote by $P_{req}(j)$ the probability that there are already j copies of P when D receives the last packet before P, $1 \leq j \leq f+1$, then we have

$$\lim_{\lambda \to \mu} P_{req}(f+1) = 1 \tag{28}$$

For a time slot, suppose some node R which carries a packet P with $SN(P) = IN(D)$ in its relay-queue specified for D, decides to conduct the relay-to-destination transmission. Without loss of generality, we assume D is selected as the receiver in the t_{th} round of probing, $1 \leq t \leq \tau$, and denote by V_i the node selected in the i_{th} round of probing, $1 \leq i < t$, $V_i \neq D$. From (28), it is easy to see that the probability that R does not carry any packet P' with $SN(P') = IN(V_i)$ $(1 \leq i < t)$ in the relay-queue for V_i, i.e., the probability p_{nc}, can be given by

$$p_{nc} = \frac{n-2-f}{n-2} \tag{29}$$

together with (27) and (5), it follows (26). Then we complete the proof for Theorem 1. ■

Lemma 8: For a MANET with the 2HR-(τ, f) routing algorithm ($1 \leq f \leq n-2$, $1 \leq \tau \leq \tau_0$), the maximum per node throughput capacity μ^* is achieved at $\tau = \tau_0$.

Proof: From (5) and (27), it is easy to see that when $\tau \in [1, \tau_0]$, μ monotonically increases with τ. Then it follows Lemma 8. ■

D. Expected End-to-End Delay of 2HR-(τ, f)

With the help of above theoretical frameworks, we proceed to analytically derive the expected end-to-end packet delay in MANETs with the 2HR-(τ, f) routing algorithm.

Definition 2: For a general packet P of the tagged flow, its end-to-end delay is defined as the time elapsed between the time slot when P is locally generated at the source node S and the time slot when P is received by the destination node D. The expected end-to-end packet delay is averaged over all packets received at the destination D in the long run.

If we denote by T_e the end-to-end delay of packet P at the tagged flow, since the end-to-end delay T_e consists of two parts, i.e., the queueing delay at the local-queue of the source node S and the packet delivery delay [12], then the expected end-to-end delay $\mathbb{E}\{T_e\}$ can be given by the following theorem.

Theorem 2: In a cell partitioned MANET where nodes move according to the i.i.d. mobility model and the 2HR-(τ, f) is adopted for packet routing, $1 \leq f \leq n-2$, $1 \leq \tau \leq \tau_0$, if the traffic flow locally generated at each source node is a Poisson stream with average input rate λ (packets/slot) $(\lambda < \mu)$, then the expected end-to-end packet delay $\mathbb{E}\{T_e\}$ can be determined as

$$\mathbb{E}\{T_e\} = \frac{\mathbb{E}\{X_D(f+1)\}}{1-\rho} \tag{30}$$

where ρ is the system load and $\rho = \lambda/\mu$.

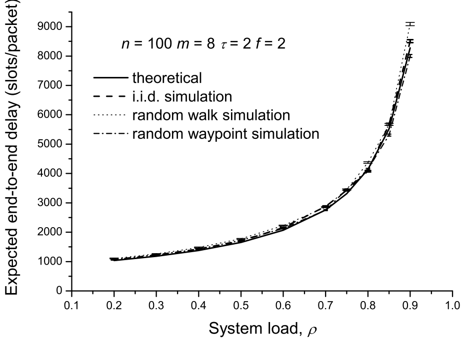
Proof: One can easily observe from Lemma 7 that under the given settings of f and τ ($1 \leq f \leq n-2$, $1 \leq \tau \leq \tau_0$), we always have $\mathbb{E}\{X_S(f+1)\} \leq \mathbb{E}\{X_D(f+1)\}$. According to Theorem 1 in [29], for a general packet P, the queueing delay and the delivery delay can be determined as 0 and $\frac{\mathbb{E}\{X_D(f+1)\}}{1-\rho}$, respectively. Summing up these two parts, it then follows (30). ■

IV. Numerical Results

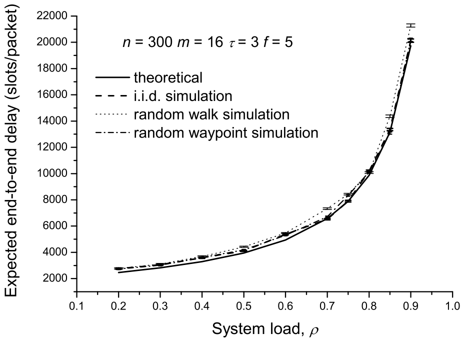
In this section, we first provide simulation results to verify the theoretical models for the per node throughput capacity and the expected end-to-end packet delay, then proceed to explore the maximum per node throughput capacity and corresponding setting of τ.

TABLE I
PARAMETER SETTINGS FOR MODEL VALIDATION

Simulation parameter	Scenario 1	Scenario 2
Number of users n	100	300
Cell partition $m \times m$	8×8	16×16
Probing round limit τ	2	3
Packet redundancy limit f	2	5
Throughput capacity μ	1.21×10^{-3}	5.07×10^{-4}



(a) Network scenario ($n = 100$, $m = 8$, $\tau = 2$, $f = 2$) with per node throughput capacity $\mu = 1.21 \times 10^{-3}$(packets/slot).



(b) Network scenario ($n = 300$, $m = 16$, $\tau = 3$, $f = 5$) with per node throughput capacity $\mu = 5.07 \times 10^{-4}$(packets/slot).

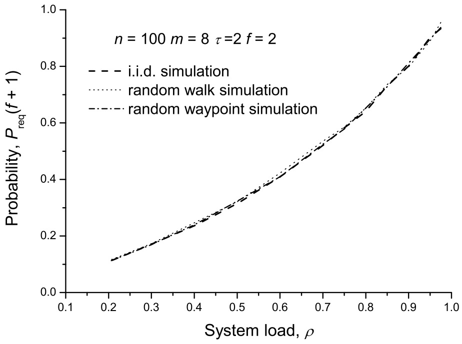
Fig. 4. Comparisons between simulation results and the theoretical ones for model validation of the per node throughput capacity and the expected end-to-end delay.

A. Simulation Settings

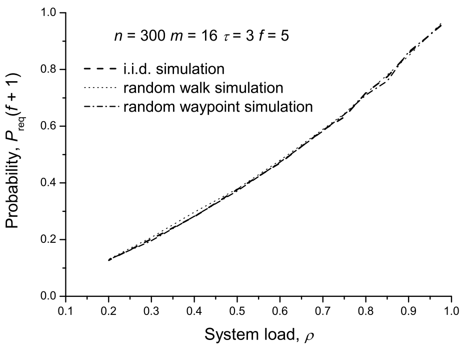
A dedicated C++ simulator was developed to simulate the packet delivery process of the proposed 2HR-(τ, f) routing algorithm, which is now available on-line at [38]. Similar to [39] the guard factor Δ was fixed as $\Delta = 1$. The traffic flow locally generated at each source node was assumed to be a Poisson stream with average input rate λ (packets/slot). Besides the i.i.d. mobility model, we also implemented the random walk and random waypoint mobility models to simulate the node movement in a MANET [13], [40].

B. Theoretical Model Validation

Extensive simulations were conducted to verify our theoretical models. Here we presented the simulation results of two network scenarios, with parameter settings listed in Table I. The simulation results of other scenarios can also be obtained by our simulator [38]. For the two scenarios listed in Table I, the comparisons between the simulation results and theoretical ones are summarized in Figs. 4 and 5. Notice that all the simulation results of the expected end-to-end delay are reported with the 95% confidence intervals.



(a) Probability $P_{\text{req}}(f+1)$ under network scenario ($n = 100$, $m = 8$, $\tau = 2$, $f = 2$).



(b) Probability $P_{\text{req}}(f+1)$ under network scenario ($n = 300$, $m = 16$, $\tau = 3$, $f = 5$).

Fig. 5. Probability $P_{\text{req}}(f+1)$ vs. system load ρ.

Figs. 4 and 5 indicate clearly that our throughput capacity model could nicely capture the throughput capacity behavior of MANETs with the 2HR-(τ, f) routing algorithm. Specifically, one can easily observe from Figs. 4a and 4b that, the simulated expected end-to-end delay there gradually increases as the system load ρ increases, and becomes extremely sensitive to the variations of ρ as ρ approaches 1. Such skyrocketing behavior of expected end-to-end delay can also serve as an intuitive validation for the throughput capacity derived by our theoretical model. Recall that $P_{req}(f+1)$ denotes the probability that there are already $f+1$ copies of a packet P in the network when its destination node receives the last packet before it. Figs. 5a and 5b show clearly that as ρ approaches 1, i.e., $\lambda \to \mu$, we have $P_{req}(f+1) \to 1$, which verifies (28) and in turn validates the throughput capacity results derived in Theorem 1. Regarding the expected end-to-end delay, one can also observe from Figs. 4a and 4b that for both network scenarios there, the theoretical expected end-to-end delay matches nicely with the simulated ones. Thus, our

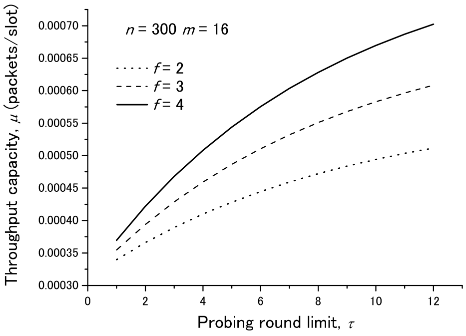
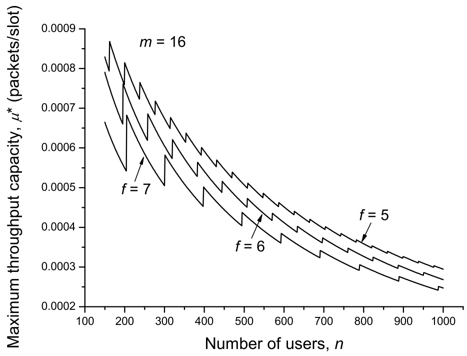
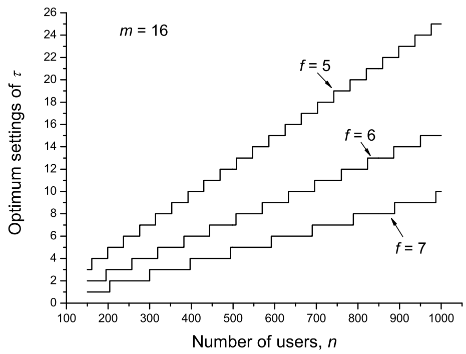


Fig. 6. Per node throughput capacity μ vs. probing round limit τ.



(a) The maximum throughput capacity μ^* vs. the number of users n.



(b) The optimum setting of τ vs. the number of users n.

Fig. 7. The maximum throughput capacity μ^* and the corresponding optimum setting of τ for networks with $m = 16$ and n varying from 150 to 1000.

theoretical models can be used to efficiently characterize the per node throughput capacity and expected end-to-end delay under the 2HR-(τ, f) routing algorithm.

It is interesting to observe from Figs. 4 and 5 that for the two network scenarios there, the simulated expected end-to-end delay and $P_{req}(f+1)$ of the 2HR-(τ, f) under the random walk and random waypoint mobility models exhibit very similar behaviors with that under the i.i.d. mobility model. Therefore, our theoretical models, although developed under the i.i.d. model, can also be used to nicely capture the network throughput capacity and expected end-to-end delay behaviors under the random walk and random waypoint mobility models.

C. 2HR-(τ, f) Throughput Capacity Analysis

Based on the theoretical model for per node throughput capacity, we first examine the impact of probing round limit τ on the per node throughput capacity μ under the 2HR-(τ, f) routing algorithm. For the network scenario $(n = 300, m = 16)$, Fig. 6 shows clearly that for each setting of f there, the per node throughput capacity μ can be significantly improved by enabling the multiple probing technique. For example, for the setting $f = 4$, the throughput capacity μ of $\tau = 5$ (resp. $\tau = 10$) is 5.44×10^{-4} (resp. 6.70×10^{-4}) (packets/slot), which is almost 1.47 (resp. 1.81) times that of $\tau = 1$ (3.70×10^{-4} (packets/slot)).

We now proceed to explore how the maximum throughput capacity μ^* and the corresponding optimum setting of τ, i.e., τ_0, vary with the number of users n. With $m = 16$ and $f = \{5, 6, 7\}$, we summarize the corresponding results in Fig. 7. One can easily observe from Fig. 7a that for each f setting there, the maximum throughput capacity μ^* vanishes quickly as the number of users n (the node density n/m^2) increases. It is also noticed that in Fig. 7a there exists a throughput capacity "jump" between some settings of n. This can be explained as follows. As shown in Fig. 7b, the optimum probing round limit τ_0 monotonically increases as n and is actually a piecewise function of n, i.e., a specific τ_0 value only applies to a small range of n. It is easy to see that in such a small range the maximum throughput capacity μ^* monotonically decreases as n increases; while as n increases beyond such range, a bigger τ_0 value is adopted and thus a higher μ^* is achieved. Thus, the throughput capacity "jump" only happens between the n values that are actually around the border line of such τ_0 ranges. A further careful observation of Fig. 7a shows that such "jump" behavior is obvious only when n is relatively small and becomes almost negligible as n increases up.

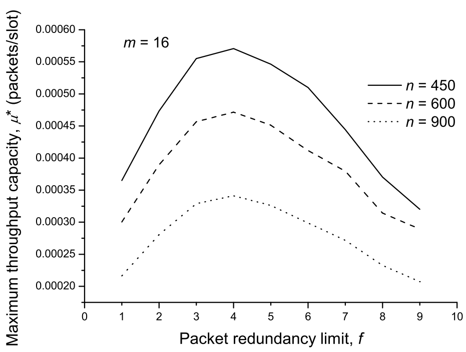


Fig. 8. The maximum throughput capacity μ^* vs. packet redundancy limit f.

Fig. 8 illustrates the relationship between the packet redundancy limit f and the maximum throughput capacity μ^*. We can see from Fig. 8 that for each n setting there, as f increases the μ^* always first increases and then decreases, and there exists an optimum setting of f such that a maximum μ^* is achieved. For example, for the case $n = 450$, 600 and 900, a maximum μ^* of 5.71×10^{-4}, 4.72×10^{-4} and 3.41×10^{-4} (packets/slot) are achieved at $f = 4$, respectively.

V. Available Throughput Capacity and Delay Analysis

A significant amount of works has been done to analyze the throughput capacity and delay performances of the two-hop relay algorithm and its variants.

A. Throughput Capacity Analysis

Since the seminal work of Grossglauser and Tse [8], a lot of order sense results have been reported for the throughput capacity of two-hop relay. It was proved that a $\Theta(1)$ per node throughput can be achieved under different mobility models, such as the i.i.d. model [8], [18], brownian mobility model [14], random walk model [13] and the restricted mobility model [15]. Ciullo *et al.* in [31] showed that when n nodes are partitioned into m clusters and each cluster-region has a circular shape of radius R, the $\Theta(1)$ throughput is also achievable in the cluster dense regime, while the throughput becomes $\Theta(mR^2/n)$ in the cluster sparse regime. Later, Li *et al.* in [35] proved that the per node throughput capacity is upper bounded by $O(n^{\beta-\alpha-1/2})$ when the network area is evenly divided into $n^{2\alpha}$ cells and each cell is further evenly divided into squares of area $n^{-2\beta}$. Recently, Perevalov and Blum examined the delay-limited throughput in [19] where it was showed that the achievable throughput grows as $d^{2/3}$ for moderate delay constraint d and scales as $\Theta(n^{-1/3})$ for a fixed delay constraint.

Recently, closed-form expressions have also been derived for the throughput capacity in MANETs. Neely and Modiano in [26] showed that in a cell partitioned MANET with fixed user density, the per node throughput capacity tends to a fixed value as the number of users scales up. Later, Urgaonkar *et al.* in [16] derived the exact network capacity and the minimum time-average power required to support it in a delay-tolerant mobile ad hoc network with Markovian mobility. The exact per node throughput capacity has also been examined in [9], [11] where a general two-hop relay with limited packet redundancy and transmission power control was considered.

B. Delay Analysis

The scaling law of packet delay under the two-hop relay algorithm has been intensively studied under different mobility models. Gamal *et al.* in [13] reported that the delay scales as $\Theta(n \log n)$ under the random walk mobility model, which was later proved to also hold under the restricted mobility model [15]. Later, Gamal *et al.* in [12] showed that under the brownian motion, the delay scales as $\Theta(n^{1/2}/v(n))$ where $v(n)$ is the velocity of mobile nodes. Lin *et al.* also considered the brownian mobility model [14], and showed that the delay is lower bounded by $\Omega(\log n/\sigma_n^2)$, where σ_n^2 is the variance parameter of the brownian motion. Sharma *et al.* in [20] showed that when the network is divided into $n^\beta \times n^\beta$ cells, the two-hop delay is $\Theta(n)$ for $\beta < 1/2$ and $\Theta(n \log n)$ for $\beta = 1/2$ under a family of mobility models. It was also proved that the delay scales as $O(\sqrt{n})$ with exact $\sqrt{n}$ packet redundancy under the i.i.d. mobility model [26], and scales as $\Theta(T_p(n)\sqrt{n}/k(n))$ under the random waypoint mobility model [27], where $T_p(n)$ is the packet transmission time and $k(n)$ is the redundancy limit.

Some closed-form results were also reported for the delay performance of two-hop relay. Groenevelt *et al.* showed that the node inter-meeting times were exponentially distributed in a sparsely distributed MANET and developed a Markov chain model for the packet delivery delay [21]. Following the same line, Hanbali *et al.* established closed-form expressions for the delivery delay of two-hop relay with limited packet redundancy [22], [23]. Later, Panagakis *et al.* in [24] analytically examined the message delivery delay where approximation expressions were provided for the cumulative distribution function of message delivery delay. More recently, Liu *et al.* derived closed-form expressions to upper bound the expected end-to-end delay [9], and characterized the exact expected end-to-end delay in MANETs with generalized transmission range and limited packet redundancy [29].

VI. Conclusion

This paper proposed a general 2HR-(τ, f) routing algorithm for efficient utilization of wireless resources in MANETs. A Markov chain theoretical framework was further developed to model the performance of the new relay algorithm, based on which closed-form expressions were derived for the per-node throughput capacity and expected end-to-end delay. Extensive simulation and theoretical studies indicate that the theoretical framework is very efficient in performance modeling for the 2HR-(τ, f) algorithm, and the new relay algorithm can significantly improve the per node throughput capacity by enabling more rounds of receiver probing. It is interesting to notice that our theoretical models for throughput capacity and expected end-to-end delay, although were developed under the i.i.d. mobility model, can also be used to nicely capture the network behaviors under the random walk and the random waypoint models as well.

It is noticed that in the proposed 2HR-(τ, f) algorithm, we considered a very simple scenario where each node is assumed to have infinite buffer space. Therefore, one of our future works is to further explore per node throughput capacity and average delay of the 2HR-(τ, f) in a more general scenario where each node has limited buffer size. Since the theoretical framework and closed-form results developed in this paper hold only for the two-hop relay routing, another future research direction is to extend the theoretical framework in this paper to analyze the throughput and delay performance for the general k-hop relay routing, $k \geq 3$.

Appendix A
Proof of the Lemmas 2 and 3

Proof of Lemma 2: For a time slot and the tagged flow, given that there are already j copies of packet P_h, the event that S will successfully deliver out a new copy for P_h occurs if and only if the following mutually exclusive sub-events happen simultaneously: the source node S is in an active cell; the destination node D is not in the one-hop neighborhood of S; s nodes out of the $n-j-1$ relays without carrying copies of P_h are in the one-hop neighborhood of S, $1 \leq s \leq n-j-1$; t nodes out of the $j-1$ relays carrying copies of P_h are in the one-hop neighborhood of S, $0 \leq t \leq j-1$; S is selected as the transmitter; S decides to conduct the source-to-relay transmission; a relay node without carrying a copy of P_h is selected as the receiver before τ rounds of probing are conducted. Then we have

$$\begin{aligned} &P_d(j) \\ &= \frac{1}{2\alpha^2}\left(1-\frac{9}{m^2}\right)\left\{\sum_{t=0}^{j-1}\sum_{s=1}^{n-j-1}\sum_{k=0}^{t+s}\binom{j-1}{t}\binom{n-j-1}{s}\right. \\ &\binom{t+s}{k}\left(\frac{1}{m^2}\right)^k\left(\frac{8}{m^2}\right)^{t+s-k}\left(1-\frac{9}{m^2}\right)^{n-2-(t+s)} \\ &\left.\cdot\frac{1}{k+1}\cdot\frac{s}{t+s}\left(1+\left(\frac{t}{t+s}\right)+\cdots+\left(\frac{t}{t+s}\right)^{\tau-1}\right)\right\} \end{aligned} \tag{31}$$

After some basic algebraic operations (4) follows and we finish the proof for Lemma 2.

Proof of Lemma 3: As the destination node D may either receive P from the source node S or receive P from some relay node, if we denote by p_{rd} the probability that D will receive P from some specific relay, say R, then we have

$$P_r(j) = p_1 + (j-1)\cdot p_{rd} \tag{32}$$

Now we proceed to derive p_{rd}. Notice that D will receive P from R if and only if the following mutually exclusive sub-events happen simultaneously: R is in an active cell; the destination node of R is not in the one-hop neighborhood of R; D is in the same cell with R or in one of its eight neighboring cells; there are k nodes other than D locating in one-hop neighborhood of R, $0 \leq k \leq n-3$; R is selected as the transmitter; R decides to conduct the relay-to-destination transmission; D is selected as the receiver in the t_{th} round of probing, $1 \leq t \leq \tau$; for the node V_i selected in the i_{th} round of probing, $1 \leq i < t$, R does not carry any packet $P^{'}$ with $SN(P^{'}) = IN(V_i)$ in its relay-queue specified for V_i. Then we have

$$\begin{aligned} p_{rd} = &\frac{1}{2\alpha^2}\left(1-\frac{9}{m^2}\right)\left\{\sum_{k=0}^{n-3}\sum_{i=0}^{k}\binom{n-3}{k}\binom{k}{i}\left(\frac{1}{m^2}\right)^i\right. \\ &\cdot\left(\frac{8}{m^2}\right)^{k-i}\left(1-\frac{9}{m^2}\right)^{n-3-k}\left(\frac{1}{m^2}\right)\frac{1}{i+2} \\ &\cdot\frac{1}{k+1}\left(1+\frac{k}{k+1}p_{nc}+\cdots+\left(\frac{k}{k+1}p_{nc}\right)^{\tau-1}\right) \\ &+\sum_{k=0}^{n-3}\sum_{i=0}^{k}\binom{n-3}{k}\binom{k}{i}\left(\frac{1}{m^2}\right)^i \\ &\cdot\left(\frac{8}{m^2}\right)^{k-i}\left(1-\frac{9}{m^2}\right)^{n-3-k}\left(\frac{8}{m^2}\right)\frac{1}{i+1} \\ &\left.\cdot\frac{1}{k+1}\left(1+\frac{k}{k+1}p_{nc}+\cdots+\left(\frac{k}{k+1}p_{nc}\right)^{\tau-1}\right)\right\} \end{aligned} \tag{33}$$

$$\begin{aligned} = &\frac{(m^2-9)^{n-2}}{2\alpha^2(n^2-3n+2)m^{2n-2}} \\ &\cdot\sum_{k=0}^{n-3}\binom{n-1}{k+2}\frac{1-(\frac{k}{k+1}p_{nc})^{\tau}}{1-\frac{k}{k+1}p_{nc}}\cdot\frac{9^{k+2}-8^{k+2}}{(m^2-9)^k} \end{aligned} \tag{34}$$

After substituting (34) into (32) it follows (5), and then we complete the proof for Lemma 3.

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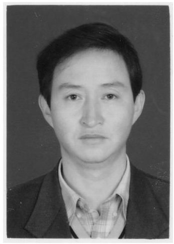

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