Optimal Forwarding Games in Mobile Ad Hoc Networks with Two-Hop f-cast Relay

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Abstract—This paper examines the optimal forwarding problem in mobile ad hoc networks (MANETs) based on a generalized two-hop relay with limited packet redundancy \( f \)-cast for packet routing. We formulate such a problem as a forwarding game, where each node \( i \) individually decides a probability \( \tau_i \), (i.e., a strategy) to deliver out its own traffic and helps to forward other traffic with probability \( 1 - \tau_i \), \( \tau_i \in [0,1] \), while its payoff is the achievable throughput capacity of its own traffic. We derive closed-form result for the per node throughput capacity (i.e., payoff function) when all nodes play the symmetric strategy profiles, identify all the possible Nash equilibria of the forwarding game, and prove that there exists a Nash equilibrium strategy profile that is strictly Pareto optimal. Finally, for any symmetric profile, we explore the possible maximum per node throughput capacity and determine the corresponding optimal setting of \( f \) to achieve it.

Index Terms—Mobile ad hoc networks, throughput capacity, Nash equilibrium, two-hop relay, packet redundancy.

I. INTRODUCTION

The mobile ad hoc network (MANET) is a highly flexible and self-autonomous wireless network architecture, where mobile nodes freely communicate with each other via wireless channels without any infrastructure support or centralized management [1], [2]. Since MANETs can be rapidly deployed, extended and reconfigured at very low cost, such networks are promising to support many important applications like disaster relief, military troop communication, last-mile internet service, etc., and thus serve as one important class of network architectures among next generation networks [3], [4].

Due to random node mobility in MANETs, the network topology may vary dramatically and no contemporaneous end-to-end path may ever exist at any given time instant [5]–[7]. The traditional route-based routing protocols such as DSR [8], AODV [9], etc., fail to work properly as they require the simultaneous availability of a number of links. As a “store-and-forward” kind of routing scheme, the two-hop relay, which relies on the mobility of nodes and sequences of their contacts to compensate for lack of continuous connectivity and thus enable messages to be delivered from end to end, becomes a promising routing protocol for the MANETs [7], [10]–[13]. The two-hop relay and its variants, simple yet efficient, have been proved to be able to provide a flexible control of both the throughput and packet delay for the challenging MANETs [7], [13]. Under such a routing protocol, the source node replicates copies of its packets to other nodes (i.e., the relay nodes) it encounters, and an intermediate relay node carrying a copy can forward the copy only to the destination node so the destination can receive a packet when meeting either the source node or one of the relay nodes. Thus, each packet travels at most two hops to reach its destination. In this paper, we consider a generalized two-hop \( f \)-cast relay (\( 2H\)-\( f \) for short), which allows each packet to be replicated to at most \( f \) distinct relay nodes.

In a general MANET, there may coexist multiple traffic flows (each corresponds to one distinct source-destination pair), so each node usually acts not only as a relay forwarding packets for other nodes, but also as a source trying to deliver out its own locally generated packets. Thus, whenever a wireless link becomes available between a node pair, the transmitting node (transmitter) likely have more incentive to deliver out a new copy for its own packet rather than forwarding a packet (if available) destined for the receiving node (receiver). Such forwarding behaviors may become more significant (and thus nodes become selfish) in MANETs where nodes not only suffer from the severe channel contention and interference issues but also operate under stringent energy consumption constraints and QoS requirements in terms of throughput and delivery delay. A significant amount of works has been done to address such selfish issue in MANETs, like the selfish node detection [14], [15], misbehavior tolerant schemes design [16], security and trust management [17], cooperation incentive stimulation [18]–[20], etc. However, the impact of selfish node behaviors on the fundamental network performances, such as the per node throughput capacity, message delivery probability and delivery delay, still remains largely unknown by now.

In this paper, we develop a novel game-theoretic framework to explicitly illustrate the relationship between forwarding behaviors of nodes and the final achievable per node throughput capacity. In particular, we study the following optimal forwarding game: in a MANET adopting the \( 2H\)-\( f \) for packet routing, each node individually decides a strategy in terms of the probability that it delivers out packets of its own flow to a node other than its destination; the target for each node here is to maximize the achievable throughput capacity of its own traffic flow, i.e., the maximum rate it can deliver traffic to its destination.
The main contributions of this paper are summarized as follows:

- For a tagged flow, we first develop a Markov chain based theoretical framework to characterize the complicated packet dispatching process at its source and the packet receiving process at its destination in the challenging MANET environment. With the help of the framework, we derive closed-form expressions for the achievable per node throughput capacity when all nodes play the symmetric strategy profiles.

- Based on the new throughput capacity results, we then develop a novel game-theoretic framework to explore the optimality properties of the forwarding game here. We identify all the possible Nash equilibrium strategy profiles, and prove that there exists a Nash equilibrium which is strictly Pareto optimal.

- Finally, for any given symmetric strategy profile, we explore the maximum possible per node throughput capacity and determine the corresponding optimal setting of \( f \) to achieve it.

**Related Works:** The optimal control of two-hop relay algorithm has been intensively explored in literature. Altman et al. in [21] provided a general framework based on fluid approximation for the optimal control of a broad class of monotone forwarding policies including the two-hop relay. Later, Altman et al. in [22] explored the general trade-offs among message delay, energy and storage when the non-monotone forwarding strategies are considered in the two-hop routing. The optimal control of two-hop relay was further explored together with forward correction and fountain codes in [23], and was explored together with Reed-Solomon type codes and network coding in [24].

Altman et al. studied the optimal stochastic control of two-hop relay in [25], where the stochastic approximation theory was employed to avoid explicit estimation of network parameters, such as the number of mobiles and their contact rates. The optimal activation and transmission control of two-hop relay for enhancing the energy saving policy were also explored in the context of delay tolerant networks (DTNs) [26]. More recently, the optimal forwarding in two-hop relay was further explored under other scenarios, like for the heterogeneous DTNs in [27], together with recovery process in [28], and in the presence of competing pure relay nodes in [29].

It is notable that all the above works [21]–[29] mainly focus on the optimization of message delivery probability under specific energy constraints and message lifetime requirement in the context of DTNs, in which the basic two-hop relay was adopted for packet routing and a wireless link becomes available whenever two nodes meet each other. Furthermore, almost all these works assumed a single source-destination pair in their analysis, where the source has only a single message to be delivered to the destination and all other nodes act as pure relay nodes. In this paper, however, we study the optimal control of the generalized 2H-f relay under more general traffic pattern in MANETs to maximize the corresponding per node throughput capacity, here the important traffic contention, medium contention and interference issues of MANETs are carefully incorporated into the study. To the best of the our knowledge, this work represents the first closed-form study of the per node throughput capacity behavior in the challenging MANETs from a game-theoretic perspective.

The remainder of the paper is organized as follows. Section II introduces the system models, the transmission-group based scheduling scheme, the 2H-f relay algorithm and the forwarding game considered in this paper. We derive closed-form expressions for the per node throughput capacity under the forwarding game in Section III, and explore the Pareto optimal Nash equilibrium and optimal setting of redundancy limit \( f \) in Section IV. Section V provides numerical results and Section VI concludes the paper.

**II. Preliminaries**

**A. System Models**

Similar to previous works [12], [13], [30]–[35], we consider a time slotted network with \( n \) mobile nodes and a unit square area evenly divided into \( m \times m \) equal cells. The nodes roam independently from cell to cell following the bi-dimensional i.i.d. mobility model widely adopted in literature [12], [13], [30], [32]–[35], where each node independently and uniformly selects a cell among the \( m^2 \) cells at the beginning of each time slot and then stays inside for the whole time slot, as illustrated in Fig. 1a.

We assume that each node conducts some form of proximity wireless communications, such that a node in one cell can only transmit to the nodes in the same cell or its eight adjacent cells (two cells are called adjacent if they share a common point). Thus, the node transmission range \( r \) can be determined as \( r = \sqrt{5}/m \). We focus on the scenario that only one-hop transmission is possible during each time slot, where the total amount of data that can be successfully transmitted per slot is fixed and assumed to be one packet here. The simple protocol model [36] is adopted to account for the interference constraint among nodes.

This paper considers the permutation traffic pattern widely adopted in previous studies [10], [33]–[35], [37], [38]. Under such traffic pattern, there are in total \( n \) distinct flows (one flow corresponds to one source-destination pair), and each node is the source of its locally generated traffic flow and at the same time the destination of the flow originated from another node. With permutation traffic, each node can be a potential relay for
other \( n - 2 \) flows (except the two flows originated from and destined for itself), so each node may simultaneously carry packets of at most \( n - 1 \) traffic flows. According to the 2H-\( f \) algorithm, a node will choose to forward traffic for other flows only when the node does not meet the destination of its own outgoing flow. In light of the fact that in the real-world MANETs some nodes may have no traffic to deliver or receive, i.e., may serve as pure relays, the per node throughput capacity derived under the permutation traffic pattern may serve as an achievable lower bound.

Remark 1: As a common partition technique for time slotted network, the general \( m \times m \) cell partition actually enables a flexible control of both the node movement speed and the node transmission range to be made for the challenging MANET, by accordingly setting the cell side length (or parameter \( m \)). Since the network topology varies dramatically and the network behavior can never be predicted under the bi-dimensional i.i.d. mobility model, the network performance analysis derived under such mobility model provides a meaningful bound in the limit of infinite mobility. Furthermore, the results in [30]–[32] indicate that the network throughput capacity derived under the i.i.d. mobility model is actually identical to the one derived under other non-i.i.d. mobility models (like the Markovian random walk model and random waypoint model) if they follow the same steady state channel distribution.

B. Transmission-Group Based Scheduling

According to the protocol interference model, multiple links can transmit simultaneously without interfering with each other if and only if they are mutually far away enough [39], [40]. To support as many simultaneous link transmissions as possible, we consider here a transmission-group based scheduling scheme similar to that of [12], [34], [35], [41].

Transmission-group: As illustrated in Fig. 1b that a transmission-group is a subset of cells, where any two of the \( m \) cells and all of them could conduct transmissions simultaneously. Based on the protocol interference model with guard factor \( \Delta \) [36], the parameter \( \alpha \) should be set properly according to \( \Delta \) to ensure the successful transmissions of all links in the same transmission-group. Notice that we assume for each node a transmission range \( r = \sqrt{8}/m \) here, by applying some derivations similar to that in [32] it is easy to see the parameter \( \alpha \) can be determined as

\[
\alpha = \min \left\{ \left(1 + \Delta \right) \sqrt{8} + 2, m \right\}
\]

(1)

Based on the parameter \( \alpha \) in (1), we can divide all \( m^2 \) cells into \( \alpha^2 \) transmission-groups such that each cell belongs to a distinct transmission-group. If the transmission-groups become active (i.e., get transmission opportunity) alternatively, then each cell becomes active in every \( \alpha^2 \) time slots. If there are more than one nodes inside an active cell, a transmitting node (transmitter) is randomly selected in a distributed way according to the method introduced in [12]. The selected node then follows the following two-hop \( f \)-cast relay for packet routing.

C. Two-Hop \( f \)-cast Relay Algorithm

A generalized two-hop \( f \)-cast (2H-\( f \)) relay is adopted here for packet routing [7], [12], [30], where \( f \) is the packet redundancy limit, \( 1 \leq f \leq n - 2 \). Under such a relay algorithm, each packet can be replicated to at most \( f \) distinct relay nodes by its source and all packets of a flow should be received in order at their destination. It is notable that with the 2H-\( f \) relay, each packet may have at most \( f + 1 \) redundant copies (including the original one at its source) in the transmission process.

Without loss of generality, henceforth we focus on a tagged flow and use \( S \) and \( D \) to denote its source and destination, respectively. To facilitate the in-order packet reception at \( D \), \( S \) labels each locally generated packet (say \( P \)) with a unique sequence number \( SN(P) \), and \( D \) maintains a request number \( RN(D) \) to denote the sequence number of the packet currently under request (in other words, \( D \) has already received all packets with sequence numbers less than \( RN(D) \)).

Every time \( S \) is selected as the transmitter in an active cell, it conducts the “source-to-destination” transmission and delivers a packet to \( D \) if \( D \) is in its one-hop transmission range (i.e., in its cell or one of its eight adjacent neighboring cells); otherwise, \( S \) randomly selects a node (say \( R \)) as the receiver among its one-hop neighborhood, and then conducts with \( R \) the “source-to-relay” transmission with probability \( \tau \) and the “relay-to-destination” transmission with probability \( 1 - \tau \), \( \tau \in [0,1] \).

D. Forwarding Game

In the above 2H-\( f \) relay, a node \( S \) tries to deliver out a new copy for the current head-of-line packet of its own flow with probability \( \tau \) while helping to forward a packet of another flow with probability \( 1 - \tau \), \( \tau \in [0,1] \). It is notable that such forwarding behavior can be nicely modeled as a two-node forwarding game (S, P) with strategy profile \( S = S_1 \times S_2 \times \ldots \times S_n \) and payoff function \( P = (\mu_1(\tau), \mu_2(\tau), \ldots, \mu_n(\tau)) \), where \( S_i \) represents the strategy set for node \( i \), and \( \mu_i(\tau) \) denotes the payoff obtained by node \( i \) under strategy profile \( \tau \in S \).

Suppose that each node \( i \in [1, n] \) individually decides its strategy as \( \tau_i \in S_i \) (i.e., it delivers out a new copy for the current head-of-line packet of its own flow with probability \( \tau_i \)) while helping to forward a packet of another flow with probability \( 1 - \tau_i \), the overall strategy profile is thus \( \tau = (\tau_1, \tau_2, \ldots, \tau_n) \), and we then define the payoff \( \mu_i(\tau) \) for node \( i \) as the achievable throughput capacity of its own flow under the strategy profile \( \tau \). For simplicity, we denote by \( \tau = (\tau_1, \ldots, \tau_n) \) a strategy profile, where \( \tau_{-i} \) is the strategies of all nodes except for node \( i \). We refer to a strategy profile with all nodes playing the same strategy as a symmetric strategy profile, or, if such a profile is a Nash equilibrium, a symmetric Nash equilibrium.

Remark 2: It is easy to see that in the above forwarding game (S, P), all nodes have identical strategy sets, i.e., \( S_i = [0,1] \) for all \( 1 \leq i \leq n \). It is also noticed that \( \mu_i(\tau_i, \tau_{-i}) = \mu_i(\tau_j, \tau_{-j}) \) for \( \tau_i = \tau_j \) and \( \tau_{-i} = \tau_{-j} \) for all \( i, j \in [1, n] \). Thus, the forwarding game (S, P) is a symmetric game with continuous strategies and continuous payoff functions.
III. THROUGHPUT CAPACITY

In this section, we first introduce some basic probabilities under the 2H-f relay, determine service times at the source and destination of a tagged flow, and then use these results to derive the per node throughput capacity (i.e. the payoff function of a node) when all nodes play the symmetric strategy profiles with \( \tau_i = \tau, 1 \leq i \leq n \).

A. Some Basic Probabilities

Based on derivations similar to that of [13], we can establish the following lemmas regarding some basic probabilities under the 2H-f relay.

Lemma 1: For a given time slot and a tagged flow, we use \( p_1 \) and \( p_2 \) to denote the probability that the source node \( S \) conducts a source-to-destination transmission and the probability that \( S \) conducts a source-to-relay or relay-to-destination transmission, respectively. Then we have

\[
p_1 = \frac{1}{\alpha^2} \left\{ \frac{9n - m^2}{n(n - 1)} - \left( \frac{m^2 - 1}{m^2} \right)^{n-1} \frac{8n + 1 - m^2}{n(n - 1)} \right\}
\]

(2)

\[
p_2 = \frac{1}{\alpha^2} \left\{ \frac{m^2 - 9}{n - 1} \left( 1 - \left( \frac{m^2 - 1}{m^2} \right)^{n-1} \right) - \left( \frac{m^2 - 9}{m^2} \right)^{n-1} \right\}
\]

(3)

Lemma 2: For a given time slot and a tagged flow, suppose the source node \( S \) is delivering copies for packet \( P \) which has already \( j \) copies in the network (including the original one at \( S \)), and the destination node \( D \) is also requesting for \( P \), i.e., the sequence number \( SN(P) \) and the request number \( RN(D) \) satisfy \( SN(P) = RN(D) \). For the next time slot, we use \( P_r(j) \), \( P_d(j) \) and \( P_s(j) \) to denote the probability that \( D \) will receive \( P \), the probability that \( S \) will successfully deliver out a copy of \( P \) to a new relay (if \( j \leq f \)) and the probability of simultaneous source-to-relay and relay-to-destination transmissions, respectively. Then we have

\[
P_r(j) = p_1 + (1 - \tau) \cdot \frac{(j - 1)p_2}{n - 2}
\]

(4)

\[
P_d(j) = \tau \cdot \frac{(n - j - 1)p_2}{n - 2}
\]

(5)

\[
P_s(j) = \frac{(\tau - \tau^2)(j - 1)(n - j - 1)(m^2 - \alpha^2)^2 \sum_{k=0}^{n-5} \binom{n - 5}{k}}{m^2 \alpha^4}
\]

\[
\cdot h(k) \left\{ \sum_{t=0}^{n-k-4} \binom{n - 4 - k}{t} h(t) \left( 1 - \frac{18}{m^2} \right)^{n-k-4-t} \right\}
\]

(6)

where

\[
h(x) = \frac{9g(x)^{x+1} - 8s(x)^{x+1}}{(x + 1)(x + 2)}
\]

(7)

B. Service Times at Source \( S \) and Destination \( D \)

To determine the service times at the source \( S \) and the destination \( D \) of a tagged flow, we first define two queues illustrated in Fig. 2. As we can see that the first queue is a local queue at \( S \), which stores the locally generated packets and operates as follows: every time a local packet arrives at \( S \), it is put to the end of the queue; every time \( S \) finishes the copy dispatching for the head-of-line packet, \( S \) takes it out of the queue and moves ahead the remaining packets behind it. Thus, the head-of-line packet of the local queue is the one for which \( S \) is currently distributing copies.

The second queue is a virtual queue defined at node \( D \), which stores only the sequence numbers of those packets not received yet by \( D \) and operates as follows: every time a packet \( P \) is moved to the head-of-line of the local queue at \( S \), the corresponding packet sequence number \( SN(P) \) is put to the end of the virtual queue; every time \( D \) receives a packet whose sequence number equals to the head-of-line entry, \( D \) moves the head-of-line entry out of the virtual queue and moves ahead the remaining entries. Thus, the head-of-line entry of the virtual queue is the sequence number of the packet that \( D \) is currently requesting for, i.e., the \( RN(D) \).

Now, the service time at \( S \) and \( D \) can be defined as follows:

Definition 1: For a packet \( P \) of the tagged flow, its service time at the source \( S \) is defined as the time elapsed between the time slot when \( S \) starts to deliver copies for packet \( P \) and the time slot when \( S \) stops distributing copies for \( P \).

Definition 2: For a packet \( P \) of the tagged flow, its service time at the destination \( D \) is defined as the time elapsed between the time slot when \( D \) starts to request for packet \( P \) and the time slot when \( D \) receives \( P \).

For a general packet \( P \) of the tagged flow, suppose that there are \( k \) copies of \( P \) in the network (including the original one
at the source $S$) when the destination node $D$ starts to request for $P$, $1 \leq k \leq f + 1$. If we denote by state $A$ the absorbing state (i.e., the termination of the service process) for $P$, then the service processes for the packet $P$ at its source $S$ and at its destination $D$ can be defined by two finite-state absorbing Markov chains shown in Fig. 3a and Fig. 3b, respectively.

Suppose that there are $k$ copies of $P$ in the network when $D$ starts to request for the packet, we denote by $X_S(k)$ and $X_D(k)$ the corresponding service time of the packet $P$ at $S$ and $D$, respectively. From the theory of Markov chain [42] we know that $X_S(k)$ is the time the Markov chain in Fig. 3a takes to become absorbed given that the chain starts from the state 1, and $X_D(k)$ is the time the Markov chain in Fig. 3b takes to become absorbed given that the chain starts from the state $k$. Based on the Markov chain in Fig. 3, we have the following results regarding the expected values $E\{X_S(k)\}$ and $E\{X_D(k)\}$ of $X_S(k)$ and $X_D(k)$.

**Lemma 3:** For a packet $P$ of the tagged flow, suppose that there are $k$ copies of $P$ in the network when the destination $D$ starts to request for $P$, $1 \leq k \leq f + 1$, then we have

1. when $0 < \tau \leq 1$,
   \[
   E\{X_S(k)\} = \begin{cases} 
   \sum_{i=1}^{k-1} \frac{1}{p_i} + \frac{1}{p_{i+1} + P_D(k)} \cdot (1 + \sum_{j=1}^{f-k} \phi_1(k, j)) & \text{if } 1 \leq k \leq f, \\
   \sum_{j=1}^{f} \frac{1}{p_{j+1}} & \text{if } k = f + 1.
   \end{cases}
   \]
   \[\text{(8)}\]

2. when $\tau = 0$, only the case $k = 1$ happens\(^1\) and we have
   \[
   E\{X_S(1)\} = E\{X_D(1)\} = \frac{1}{p_1}
   \]
   \[\text{(9)}\]

**Remark 3:** The proofs of above lemmas are similar to that in [12]. The basic idea behind the proof of Lemma 3 is to show that for the Markov chains in Figs. 3a and 3b, the corresponding absorbing time can be actually derived based on recursive calculation technique.

**C. Per Node Throughput Capacity**

With the help of the results in Lemmas 3 and 4, we can establish the following theorem on the per node throughput capacity.

**Theorem 1:** For the cell partitioned MANET considered in this paper, where each node moves according to the i.i.d. mobility model and follows the 2H-f relaying algorithm for packet routing, if we denote by $\mu(\tau)$ the per node throughput capacity (i.e. the payoff function of a node) when all nodes play the symmetric strategy profile $\tau$ with $\tau_i = \tau$, $1 \leq i \leq n$, then we have

\[
\mu(\tau) = \begin{cases} 
\min\{G_1(\tau, f), G_2(\tau, f)\} & \text{if } 0 < \tau < 1, \\
p_1 & \text{if } \tau = 0 \text{ or } \tau = 1.
\end{cases}
\]
\[\text{(15)}\]

where
\[
G_1(\tau, f) = p_1 + \frac{(1 - \tau) \cdot f \cdot p_2}{n - 2}
\]
\[\text{(16)}\]

\[
G_2(\tau, f) = \frac{p_1 \cdot \tau \cdot p_2}{1 + \frac{1}{\prod_{i=1}^{n} \frac{1}{(n - t - 1) \cdot \tau \cdot p_2}}}
\]
\[\text{(17)}\]

**Proof:** We first consider the case $0 < \tau < 1$. Lemmas 3 and 4 indicate that under the 2H-f relay algorithm, the parameter $k$ is automatically updated from packet to packet to adjust to the service rates at $S$ and $D$. Based on this intrinsic feature of automatic updating for parameter $k$, we can model the packet delivery process of the tagged flow as an automatic feedback control system [32]. Thus, according to Theorem 1 in [32], the per node throughput capacity $\mu(\tau)$ here is determined as

\[
\mu(\tau) = \min\left\{\frac{1}{E\{X_D(f + 1)\}}, \frac{1}{E\{X_S(1)\}}\right\}
\]
\[\text{(18)}\]

The (15) then follows after substituting (8) and (9) into (18).

Regarding the cases $\tau = 0$ and $\tau = 1$, it’s easy to see that for a tagged flow, its destination node $D$ will receive every packet only from its source node $S$. Thus, we have $\mu(\tau) = p_1$.

**IV. Optimality**

Based on the payoff function of a node (i.e. per node throughput capacity) derived in Section III, this section first identifies all the possible Nash equilibrium strategy profiles, and then proves that there exists a Nash equilibrium strategy profile which is strictly Pareto optimal. Finally, we explore the maximum possible per node throughput capacity for any given symmetric strategy profile and determine the corresponding optimal setting of $f$ to achieve it.

\[^1\]Under the extreme case $\tau = 0$, each packet has only one copy (the original one at its source) when its destination starts to request for it, and the destination can only receive the packet directly from its source.
A. Nash Equilibrium and Pareto Optimality

Recall that in this paper we consider a forwarding game \((S, P)\) where each node \(i\) individually selects a strategy \(\tau_i\) with the aim of maximizing its own payoff \(\mu_i(\tau_i, \tau_{-i})\). We first establish the following two lemmas, which will help us to identify all the Nash equilibria and also the Pareto optimal Nash equilibrium for the above forwarding game.

Lemma 5: Given that all nodes play the symmetric strategy profile \(\tau\) with \(\tau_i = \tau, \ i \in [1, n]\). For any given packet redundancy limit \(f \in [1, n-2]\) and the two functions \(G_1(\tau, f)\) and \(G_2(\tau, f)\) defined in (16)-(17), there exists a unique \(\tau^* \in (0, 1)\) such that

\[
\tau^* = \arg_{0<\tau<1} \{ G_1(\tau, f) = G_2(\tau, f) \} \quad (19)
\]

and

\[
G_1(\tau, f) > G_2(\tau, f) \quad \text{for} \ \forall \tau \in (0, \tau^*) \quad (20)
\]

\[
G_1(\tau, f) < G_2(\tau, f) \quad \text{for} \ \forall \tau \in (\tau^*, 1) \quad (21)
\]

Proof: We first prove that for any given \(f \in [1, n-2]\) there exists a unique \(\tau^*\) that satisfies (19). Notice that

\[
G_1(\tau, f)|_{\tau \to 0} = p_1 + \frac{f}{n-2} \cdot p_2 \quad (22)
\]

\[
G_2(\tau, f)|_{\tau \to 0} = p_1 \quad (23)
\]

\[
G_1(\tau, f)|_{\tau \to 1} = p_1 \quad (24)
\]

\[
G_2(\tau, f)|_{\tau \to 1} > p_1 \quad (25)
\]

where (25) follows because \(G_2(\tau, f) = \frac{1}{2(1-\tau)}\) and \(\mathbb{E}\{X_S(1)\}|_{\tau = 1} < \frac{1}{2}\), as can be proved recursively based on the Markov chain in Fig. 3a.

It’s easy to see that for any given \(f, G_1(\tau, f)\) monotonically decreases as \(\tau\) increases, while the Markov chain in Fig. 3a indicates that as \(\tau\) increases, \(\mathbb{E}\{X_S(1)\}\) monotonically decreases and thus \(G_2(\tau, f)\) monotonically increases. Combining the results in (22), (23), (24) and (25), we can see that only a unique solution \(\tau^*\) exists for (19).

Since for a fixed \(f, G_1(\tau, f)\) (resp. \(G_2(\tau, f)\)) monotonically decreases (increases) as \(\tau\) increases and there is a unique solution to (19), we can see from (22) and (23) that \(G_1(\tau, f) \geq G_2(\tau, f)\) if 0 < \(\tau < \tau^*\) and we can see from (24) and (25) that \(G_1(\tau, f) \leq G_2(\tau, f)\) if \(\tau^* < \tau < 1\). This finishes the proof for Lemma 5.

Lemma 6: For any given packet redundancy limit \(f \in [1, n-2]\), if all nodes play symmetric strategy profiles in the forwarding game \((S, P)\), then each node \(i (i \in [1, n])\) can obtain the maximum payoff by selecting the strategy \(\tau_i = \tau^*\), where \(\tau^*\) is given by (19).

Proof: By combining the per node throughput capacity result (15) in Theorem 1 and (19), (20) and (21) in Lemma 5, it follows that each node \(i\) obtains the maximum payoff by choosing \(\tau_i = \tau^*\).

As indicated in Remark 2 that the forwarding game \((S, P)\) considered in this paper is a symmetric game with continuous strategies and continuous payoff functions, and thus there is often a Nash equilibrium (i.e., the symmetric Nash equilibrium) where all nodes are playing the same strategy. The following theorem identifies the possible Nash equilibria of the forwarding game.

Theorem 2: For the forwarding game \((S, P)\), any symmetric strategy profile \(\tau\) with \(\tau_i = \tau, \ i \in [1, n]\), is a Nash equilibrium if \(\tau^* \leq \tau \leq 1\), where \(\tau^*\) is determined by (19).

Proof: According to the definition of Nash equilibrium, we only need to prove that when \(\tau^* \leq \tau \leq 1\),

\[
\forall \ i \in [1, n], \ \tau_i' \in S_i, \ \tau_i' \neq \tau ; \ \mu_i(\tau, \tau_{-i}) \geq \mu_i(\tau_i', \tau_{-i})
\]

(26)

We first prove the case that \(\tau^* \leq \tau < 1\). For a tagged node \(i\), from its payoff function (15) we can see that the function \(G_1(\tau, f)\) is only affected by the strategies of all other nodes (i.e., \(\tau_{-i}\)), while the function \(G_2(\tau, f)\) is affected only by the strategy of node \(i\) (i.e., \(\tau_i\)). With a little abuse of the notations, we use \(G_1(\tau_{-i}, f)\) and \(G_2(\tau_i, f)\) to represent these two functions, respectively.

When all nodes except the node \(i\) play the strategy \(\tau\), \(\tau^* \leq \tau < 1\), the function \(G_1(\tau, f)\) is always fixed as \(G_1(\tau_{-i}, f)\). Suppose that the node \(i\) selects another strategy \(\tau_i' \neq \tau\), only \(G_2(\tau_i, f)\) is updated to \(G_2(\tau_{-i}, f)\). Since \(\tau_i' \neq \tau\), we may have \(G_2(\tau_i', f) < G_1(\tau_{-i}, f)\) or \(G_2(\tau_i', f) \geq G_1(\tau_{-i}, f)\). From the payoff function (15) we can see that for the first case, we have \(\mu_i(\tau, \tau_{-i}) > \mu_i(\tau_i', \tau_{-i})\); while for the latter case, we have \(\mu_i(\tau, \tau_{-i}) = \mu_i(\tau_i', \tau_{-i})\). Thus, we can see that (26) holds and the node \(i\) cannot profit from its unilateral deviation in the strategy profile \(\tau\).

Regarding the case that \(\tau = 1\), since each node only delivers out its own traffic and no one will act as a relay, we always have \(\mu_i(\tau, \tau_{-i}) = \mu_i(\tau_i', \tau_{-i}) = p_1\) for all \(i \in [1, n]\). Then we complete the proof for Theorem 2.

Among all the Nash equilibria identified in Theorem 2, the following theorem shows that there exists a unique Nash equilibrium which is strictly Pareto optimal.

Theorem 3: The Nash equilibrium \(\tau^* = (\tau^*, \tau^*, \ldots, \tau^*)\), where \(\tau^*\) is determined as (19), is strictly Pareto optimal.

Proof: We can see from Lemma 6 and Theorem 2 that among all the Nash equilibria, each node \(i \in [1, n]\) can obtain its maximum payoff (i.e., the maximum throughput capacity) only with the Nash equilibrium \(\tau^* = (\tau^*, \tau^*, \ldots, \tau^*)\). Thus, the Nash equilibrium strategy profile \(\tau^*\) is strictly Pareto optimal.

Remark 4: All the Nash equilibrium strategy profiles identified in Theorem 2 are weak Nash equilibria. The result in Theorem 3 indicates that for the concerned MANET operating under the 2H-f relay, each node there should adopt the optimal forwarding strategy \(\tau^*\), i.e., the Pareto optimal Nash equilibrium, to ensure the optimum per node throughput capacity. Notice that in the forwarding game \((S, P)\), we consider a fixed ad hoc environment where no nodes will join or secede from the network. Based on our game-theoretic framework, therefore, each node needs only to acquire the network size \(n\) (i.e., the number of nodes) before it is able to decide individually the optimal forwarding strategy \(\tau^*\). In
the implementation of the forwarding game, each node may employ a network size estimation technique to acquire (or approximate) the network size, such as the capture-recapture method [43], the random tour and gossip-based aggregation algorithms [45], etc.

B. Optimal Setting of $f$

This section further explores the optimal setting of packet redundancy limit $f$ for the maximization of per node throughput (15) under any symmetric strategy profile $\tau$ with $\tau_i = \tau$, $0 < \tau < 1$, $i \in [1,n]$.

Theorem 4: For the forwarding game $(S,P)$ and any given symmetric strategy profile $\tau$ with $\tau_i = \tau$, $0 < \tau < 1$, $i \in [1,n]$, there exists an optimal packet redundancy limit $f^*$ such that each node achieves the maximum per node throughput capacity $\mu^*$ determined as:

(1) if $0 < \tau \leq \frac{1}{n-1}$,

\[ f^* = 1 \quad (27) \]
\[ \mu^* = p_1 + \tau \cdot p_2 \quad (28) \]

(2) if $\frac{1}{n-1} < \tau \leq \tau_1$,

\[ f^* = \begin{cases} f_0 & \text{if } G_1(\tau, f_0) \geq G_2(\tau, f_1) , \\ f_1 & \text{if } G_2(\tau, f_1) > G_1(\tau, f_0). \end{cases} \quad (29) \]
\[ \mu^* = \max\{G_1(\tau, f_0), G_2(\tau, f_1)\} \quad (30) \]

(3) if $\tau_1 < \tau < 1$,

\[ f^* = n - 2 \quad (31) \]
\[ \mu^* = p_1 + (1 - \tau) \cdot p_2 \quad (32) \]

where

\[ f_0 = \max\{f \in [1,n-2] \mid G_1(\tau, f) \leq G_2(\tau, f)\} \quad (33) \]
\[ f_1 = \min\{f \in [1,n-2] \mid G_2(\tau, f) \leq G_1(\tau, f)\} \quad (34) \]
\[ \tau_1 = \arg_{0 < \tau < 1} \{G_1(\tau, n-2) = G_2(\tau, n-2)\} \quad (35) \]

Proof: Since the equation (19) in Lemma 5 holds for any $f \in [1,n-2]$, for the specific setting of $f = n-2$ we can certainly determine a value $\tau_1$ according to (35). Based on (20) and (21), we know that

\[ G_1(\tau, n-2) > G_2(\tau, n-2) \text{ if } 0 < \tau < \tau_1 \quad (36) \]
\[ G_1(\tau, n-2) < G_2(\tau, n-2) \text{ if } \tau_1 < \tau < 1 \quad (37) \]

Similarly, for the setting $f = 1$, we can also determine a value $\tau_0$ as $\tau_0 = \arg_{0 < \tau < 1} \{G_1(\tau, 1) = G_2(\tau, 1)\} = \frac{1}{n-1}$, where

\[ G_1(\tau, 1) > G_2(\tau, 1) \text{ if } 0 < \tau < \tau_0 \quad (38) \]
\[ G_1(\tau, 1) < G_2(\tau, 1) \text{ if } \tau_0 < \tau < 1 \quad (39) \]

We now show that $\tau_1 > \frac{n-2}{n-1} > \tau_0$. Notice that

\[ G_1(\frac{n-2}{n-1}, n-2) = p_1 + \frac{p_2}{n-1} \quad (40) \]

and

\[ G_2(\frac{n-2}{n-1}, n-2) < (p_1 + P_d(f))_{|f=n-2} = p_1 + \frac{p_2}{n-1} \quad (41) \]
\[ \text{where (41) follows because that } G_2(\tau, f) = \frac{1}{\mathbb{E}\{X_1(1)\}} \text{ and } \mathbb{E}\{X_S(1)\} > \frac{1}{\mu^*}, \text{ Combining (40) and (42), we then have } G_1(\tau, n-2) > G_2(\tau, n-2) \text{ for the setting } \tau = \frac{n-2}{n-1}. \text{ Thus, we can see from (36) that } \tau_1 > \frac{n-2}{n-1} > \tau_0. \]

We first consider the case that $0 < \tau \leq \tau_0 = \frac{1}{n-1}$. Notice that for any given $\tau \in (0,1)$, as $f$ increases, $G_1(\tau, f)$ monotonically increases while $G_2(\tau, f)$ monotonically decreases. From (36) and (38), we can see that if $0 < \tau \leq \tau_0$, we have $G_1(\tau, f) \geq G_2(\tau, f)$ for any $f \in [1,n-2]$. Thus, combining with (15), the maximum throughput capacity here is determined as $G_2(\tau, 1)$, so (27) and (28) follow.

Regarding the case that $\tau_0 < \tau \leq \tau_1$, we can see from (36) and (39) that we can determine two values $f_0$ and $f_1$ of $f$ according to (33) and (34), respectively. Notice that for any $f \in (f_1, f_0)$ we always have $G_1(\tau, f) < G_2(\tau, f)$, and for any $f \in (f_1, n-2]$ we always have $G_2(\tau, f) < G_1(\tau, f)$. Thus, it is easy to see that (29) and (30) follow.

Finally, for the case that $\tau_1 < \tau < 1$, we can see from (37) and (39) that $G_1(\tau, f) < G_2(\tau, f)$ for any $f \in [1,n-2]$. Thus, the maximum per node throughput capacity can be determined as $G_1(\tau, n-2)$, and (31) and (32) then follow.

V. Numerical Results

Based on the above theoretical framework, this section provides numerical results to illustrate the network performance under a symmetric forwarding game. The guard factor $\Delta$ is fixed as $\Delta = 1$ here, so the parameter $\alpha$ for transmission-group is determined as $\alpha = \min\{8, m\}$.

A. Per Node Throughput Capacity $\mu(\tau)$ vs. $(\tau, f)$

We first explore how the per node throughput capacity $\mu(\tau)$ (i.e., the payoff of each node) varies with both the packet redundancy limit $f$ and a strategy profile $\tau$ with $\tau_i = \tau$ for all $i \in [1,n]$. For a network of $n = 100$ and $m = 16$, we summarize the corresponding results in Fig. 4. One can easily observe from Fig. 4a that for each setting of $f \in [1,32]$ there, we can always find a corresponding optimum strategy $\tau$, i.e., the strictly Pareto optimal Nash equilibrium, to achieve the maximum per node throughput capacity. For example, as shown in Fig. 4b, when $f = 5$, 12 and 20, each node can obtain a maximum throughput capacity of $8.883 \times 10^{-4}$, $9.023 \times 10^{-4}$ and $7.82 \times 10^{-4}$ with the strategies of $\tau = 0.26$, 0.68 and 0.88, respectively. Similarly, for each strategy $\tau$, we can also find an optimum setting of $f$ such that the corresponding per node throughput capacity is maximized. For example, as shown in Fig. 4c, when $\tau = 0.40$, 0.60 and 0.80, the maximum per node throughput capacity of $9.373 \times 10^{-4}$, $9.253 \times 10^{-4}$ and $8.128 \times 10^{-4}$ are obtained by settings $f = 7$, 10 and 16, respectively. The results in Fig. 4a also indicate clearly that for the concerned network there does exist an optimum combination $(\tau = 0.48, f = 8)$ of strategy profile $\tau$ and packet redundancy limit $f$ at which the global maximum per node throughput capacity $9.450 \times 10^{-4}$ is achieved.
there always exists a corresponding optimum strategy \( \tau^* \) that is unique and can be determined for each network size. One can easily observe from Fig. 5a that for each setting of \( f \) there, \( \tau^* \) monotonically decreases as \( n \) increases. This result indicates for a larger network, each node there should forward packets for others (rather than for itself) with a higher probability so as to maximize the throughput capacity of its own flow. It’s also notable that for a network with fixed size \( n \), the optimal strategy value \( \tau^* \) for a larger \( f \) also becomes larger, which implies that for a fixed network size, if more packet redundancy is allowed for each packet, each node should adopt a higher probability to deliver its own packets.

It is interesting to observe from Fig. 5b that for a given setting of \( f \) there, we can always determine a suitable network density (defined as \( n/m^2 \)) at which the corresponding \( \mu(\tau^*) \) is the maximum among all network densities. Actually, such “optimal” network density increases as \( f \) increases. For example, for the settings of \( f = 10, f = 15 \) and \( f = 25 \), the corresponding maximum \( \mu(\tau^*) \) is achieved at the network densities of 0.161, 0.214 and 0.269, respectively. Therefore, we can see that given the per node power constraint (related to \( f \)), the network density should be carefully designed so as to maximize the per node throughput capacity there. It is also notable that different from what is observed in Fig. 5a, \( \mu(\tau^*) \) varies differently with \( f \) under different region of \( n \). In particular, when \( 50 \leq n \leq 199 \), we have \( \mu(\tau^*)|_{f=10} > \mu(\tau^*)|_{f=15} > \mu(\tau^*)|_{f=25} \); when \( 200 \leq n \leq 349 \), we have \( \mu(\tau^*)|_{f=15} = \mu(\tau^*)|_{f=25} > \mu(\tau^*)|_{f=10} \); when \( 350 \leq n \leq 536 \), we have \( \mu(\tau^*)|_{f=15} > \mu(\tau^*)|_{f=25} > \mu(\tau^*)|_{f=10} \); when \( 537 \leq n \leq 1000 \), we have \( \mu(\tau^*)|_{f=25} > \mu(\tau^*)|_{f=15} > \mu(\tau^*)|_{f=10} \).

C. Optimum Setting \( f^* \) and \( \mu^* \) vs. \( n \)

We further show in Fig. 6 that when all nodes follow a given symmetric strategy profile \( \tau_i = \tau \) for all \( i \in [1, n] \), how the optimum setting \( f^* \) and the corresponding maximum throughput capacity \( \mu^* \) vary with network size \( n \). One can easily observe from Fig. 6a that \( f^* \) is a non-decreasing piecewise function of \( n \) for each setting of \( \tau \) there, i.e., under a fixed \( \tau \), a specific value of \( f^* \) can only apply to a small range of \( n \). It is also noticed that for a fixed \( n \), \( f^* \) of a bigger \( \tau \) is also bigger, i.e., if a higher probability is adopted for each node to deliver out its own packets, more redundancy should be allowed for each packet such that the

Fig. 5. Strictly Pareto optimal Nash equilibrium strategy \( \tau^* \) and the corresponding optimum per node throughput capacity \( \mu(\tau^*) \).

B. Pareto Optimal Nash Equilibrium \( \tau^* \) and \( \mu(\tau^*) \) vs. \( n \)

Notice that for a network with a fixed \( f \in [1, n - 2] \), there always exists a corresponding optimum strategy \( \tau^* \) (i.e., the Pareto optimal Nash equilibrium strategy) by which each node obtains the optimum throughput capacity \( \mu(\tau^*) \). Fig. 5 illustrates how such \( (\tau^*, \mu(\tau^*)) \) vary with network size \( n \) for the settings of \( m = 24 \) and \( f \in \{10, 15, 25\} \). One can easily observe from Fig. 5a that for each setting of \( f \) there, \( \tau^* \) monotonically decreases as \( n \) increases. This result indicates for a larger network, each node there should forward packets for others (rather than for itself) with a higher probability so as to maximize the throughput capacity of its own flow. It’s also notable that for a network with fixed size \( n \), the optimal strategy value \( \tau^* \) for a larger \( f \) also becomes larger, which implies that for a fixed network size, if more packet redundancy is allowed for each packet, each node should adopt a higher probability to deliver its own packets.

It is interesting to observe from Fig. 5b that for a given setting of \( f \) there, we can always determine a suitable network density (defined as \( n/m^2 \)) at which the corresponding \( \mu(\tau^*) \) is the maximum among all network densities. Actually, such “optimal” network density increases as \( f \) increases. For example, for the settings of \( f = 10, f = 15 \) and \( f = 25 \), the corresponding maximum \( \mu(\tau^*) \) is achieved at the network densities of 0.161, 0.214 and 0.269, respectively. Therefore, we can see that given the per node power constraint (related to \( f \)), the network density should be carefully designed so as to maximize the per node throughput capacity there. It is also notable that different from what is observed in Fig. 5a, \( \mu(\tau^*) \) varies differently with \( f \) under different region of \( n \). In particular, when \( 50 \leq n \leq 199 \), we have \( \mu(\tau^*)|_{f=10} > \mu(\tau^*)|_{f=15} > \mu(\tau^*)|_{f=25} \); when \( 200 \leq n \leq 349 \), we have \( \mu(\tau^*)|_{f=15} = \mu(\tau^*)|_{f=25} > \mu(\tau^*)|_{f=10} \); when \( 350 \leq n \leq 536 \), we have \( \mu(\tau^*)|_{f=15} > \mu(\tau^*)|_{f=25} > \mu(\tau^*)|_{f=10} \); when \( 537 \leq n \leq 1000 \), we have \( \mu(\tau^*)|_{f=25} > \mu(\tau^*)|_{f=15} > \mu(\tau^*)|_{f=10} \).

C. Optimum Setting \( f^* \) and \( \mu^* \) vs. \( n \)

We further show in Fig. 6 that when all nodes follow a given symmetric strategy profile \( \tau_i = \tau \) for all \( i \in [1, n] \), how the optimum setting \( f^* \) and the corresponding maximum throughput capacity \( \mu^* \) vary with network size \( n \). One can easily observe from Fig. 6a that \( f^* \) is a non-decreasing piecewise function of \( n \) for each setting of \( \tau \) there, i.e., under a fixed \( \tau \), a specific value of \( f^* \) can only apply to a small range of \( n \). It is also noticed that for a fixed \( n \), \( f^* \) of a bigger \( \tau \) is also bigger, i.e., if a higher probability is adopted for each node to deliver out its own packets, more redundancy should be allowed for each packet such that the
per node throughput capacity is maximized, which is similar to what is observed in Fig. 5a. A further careful observation of Fig. 6a indicates that \( f^* \) becomes more sensitive to the variation of \( n \) when a larger value of \( \tau \) is considered, which implies that a setting of \( f^* \) can be applied to a wider range of \( n \) if the network operates under a smaller value of \( \tau \).

The results in Fig. 6b show that for all the three settings of \( \tau \) there, the general trends of \( \mu^* \) with \( n \) are actually very similar, and they all diminish quickly as \( n \) increases. However, it’s interesting to observe that under any fixed setting of \( n \) here, we always have \( \mu^*|\tau=0.50 > \mu^*|\tau=0.30 > \mu^*|\tau=0.80 \). A further careful observation of Fig. 6b indicates that, for each setting of \( \tau \), \( \mu^* \) varies non-monotonically with \( n \), which is especially the case when \( n \) is relatively small. Specifically, \( \mu^* \) has two different kinds of rise up behaviors as \( n \) increases. The first kind happens in the range of \( n \) where a specific \( f^* \) applies. For example, for the setting \( \tau = 0.50 \) (resp. \( \tau = 0.30 \)), a maximum \( \mu^* \) of \( 6.478 \times 10^{-4} \) (resp. \( 6.246 \times 10^{-4} \)) is achieved when \( n \in [140, 167] \) (resp. \( n \in [124, 164] \)) where the optimum setting \( f^* = 11 \) (resp. \( f^* = 7 \)) is adopted. Therefore, we can see that for a fixed combination of \( \tau \) and \( f \), there may exist an optimum value of \( n \) to maximize the per node throughput capacity. The second kind of rise up behavior happens near the border of two ranges of network size \( n \) where different settings of \( f^* \) are adopted. For example, for the setting of \( f^* = 11 \) with \( \tau = 0.50 \) (resp. the setting of \( f^* = 7 \) with \( \tau = 0.30 \)), the \( \mu^* \) also rises up as \( n \) increases slightly beyond the border value of \( n = 167 \) (resp. \( n = 164 \)). Indeed, such fluctuation behavior of \( \mu^* \) near the border values can be attributed to the following reason: as a specific value of \( f^* \) can only apply to a small range of \( n \), when \( n \) increases slightly beyond the border value a new and bigger \( f^* \) is adopted; since the number of nodes \( n \) is small and the difference between \( n \) is also very small, the adoption of bigger \( f^* \) (i.e., more redundant copies) dominates the impact on \( \mu^* \) and thus we have a sharp rise up of \( \mu^* \).

\( \textbf{VI. Conclusion} \)

This paper developed a game-theoretic framework for investigating the optimal forwarding control issue of the general 2H-\( f \) relay algorithm in MANETs. For one MANET with any specified setting of packet redundancy limit \( f \), this framework helps us to identify the optimal forwarding strategy that each node should adopt to ensure the optimum per node throughput capacity. On the other hand, for a MANET operating under any specified forwarding strategy (including all the possible Nash equilibrium forwarding strategies), this framework enables an optimal setting of \( f \) to be determined for the maximization of per node throughput capacity.

Our results in this paper indicate that for a fixed setting of packet redundancy limit \( f \), we can always find a suitable network size (node density) such that the corresponding Pareto optimal throughput is the maximum among all network sizes. It is also interesting to notice that different from the general intuitive expectation that as network size becomes larger, each node should adopt a higher probability to forward packets for other flows (rather than for its own flow) to maximize the throughput capacity of its own flow.

The game-theoretic framework in this paper was developed under the assumption of a fixed network environment, where all nodes will move around in the network area and no new nodes will join the network. Therefore, one of our future research directions is to extend the theoretical framework in this paper to explore the optimal forwarding strategies of a varying ad hoc environment, where nodes can join or secede from network arbitrarily. Notice also that the closed-form results of per node throughput capacity derived in this paper only hold for the simple network scenario with i.i.d. node distribution, so our another future research direction is to develop theoretical models for other real environment with more practical node distributions, like the correlated distribution, the clustered distribution and the reference point distribution.

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